Distributed Fault Detection for Interconnected Second-Order Systems with Applications to Power Networks

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***Abstract*—Observers for distributed fault detection of interconnected second-order linear time invariant systems is studied. Particularly, networked systems under consensus protocols are considered and it is proved that for these systems one can construct a bank of so-called unknown input observers, and use their output to detect and isolate possible faults in the network. The application of this family of fault detectors to power networks is presented.**

I. Introduction

Critical infrastructures such as power grids, water distribution networks, and transport systems are examples of cyber-physical systems. These systems consist of large-scale physical processes monitored and controlled by SCADA (supervisory control and data acquisition) systems running over a heterogeneous set of communication networks and computers. Although the use of such powerful software systems adds flexibility and scalability, it also increases the vulnerability to hackers and other malicious entities who may perform cyber attacks through the IT systems [1], [2]. Several security breaches have been recently announced [3], [4].

A holistic approach to security of SCADA systems is important because of the complex coupling between the physical process and the distributed software system. Unfortunately a theory for such system security lacking. Increasing the security by adding encryption and authentication schemes helps to prevent some cyber attacks by making them harder to succeed but it would be a mistake to rely solely on such methods, as it is well-known that the overall system is not secured because some of its components are. A method to increase security of networked control systems involve the design of control algorithms that are robust to the effects of cyber attacks [5], [6], [7], [8] and monitoring schemes to detect anomalies in the system caused by attacks [9]. This paper focus on the latter and uses fault detection and isolation (FDI) to design a distributed FDI scheme for a network of interconnected second-order linear systems.

There are various ways to detect and isolate a fault in a system [10], [11], [12], [13]. Observer based approaches have been well studied and some of these methods have been proposed for power systems [14], [15]. However, distributed FDI for systems comprised of a network of autonomous nodes still lacks a thorough theory. A relevant and interesting result is presented in [9] where the authors have proposed a discrete time algorithm to detect the misbehaving node in a network of nodes with single integrator dynamics. Another result related to this work is [16] where the possibility of detecting faults by coordinating certain movements in the formation is shown.

In this paper we consider the problem of distributed fault detection and isolation in a network of nodes with double integrator dynamics seeking to reach consensus. To achieve this goal, we design a bank of continuous time unknown input observers (UIO) in each node, which then monitors its own neighborhood. The existence of such observers is established for two different consensus protocols, and some infeasibility results are provided. As an illustrative example, the application of the proposed method to FDI in power networks is presented.

The outline of the paper is as follows. In the next section the problem is formulated. In Section III we introduce the UIO that we use to obtain the main result of this paper. In Section IV, we propose a solution to the problem posed in Section II. In Section V the application of the method on an illustrative 9 bus power grid is studied. Conclusions and future remarks come in the last section.

II. Problem Formulation

Consider a network of N interconnected nodes and let $\mathcal{G}(\mathcal{V},\mathcal{E},\mathcal{A})$ be the underlying graph of this network, where $\mathcal{V}=\{i\}_1^N$ is the vertex set with $i\in\mathcal{V}$ corresponding to node i, $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ is the edge set of the graph, and $\mathcal{A}\in\mathbb{R}^{N\times N}$ is the weighted adjacency matrix with nonnegative entries. The undirected edge $\{i,j\}$ is incident on vertices i and j if nodes i and j share a communication link, in which case the corresponding entry in the adjacency matrix $[\mathcal{A}]_{ij}$ is positive and reflects the edge weight. The out-degree of node i is $\deg(i)=\sum_{j\in N_i}[\mathcal{A}]_{ij}$, where $N_i=\{j\in\mathcal{V}:\{i,j\}\in\mathcal{E}\}$ is the neighborhood set of i. The degree matrix $\Delta(\mathcal{G})\in\mathbb{R}^{N\times N}$ is a diagonal matrix defined as

$$[\Delta]_{ij}=\begin{cases}\deg(i) & ,\ i=j\\ 0 & ,\ i\neq j\end{cases}.$$

The weighted Laplacian of $\mathcal{G}$ is defined as $\mathcal{L}(\mathcal{G})=\Delta-\mathcal{A}$.

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Each node i is assumed to have double integrator dynamics

$$\dot{\xi}_i(t) = \zeta_i(t) \tag{1a}$$
$$\dot{\zeta}_i(t) = u_i(t), \tag{1b}$$

where ξ_i, ζ_i, and u_i are scalars and u_i is the control law

$$u_i(t) = \sum_{j \in N_i} \left[w_{ij}(\xi_j(t) - \xi_i(t)) + (\alpha_{ij}\zeta_j(t) - \beta_i \zeta_i(t)) \right]. \tag{2}$$

We say that node k is faulty if for some functions $f_{\xi k}(t)$ and $f_{\zeta k}(t)$ not identical to zero it holds that

$$\begin{aligned} \dot{\xi}_k(t) &= \zeta_k(t) + f_{\xi k}(t) \\ \dot{\zeta}_k(t) &= u_k(t) + f_{\zeta k}(t). \end{aligned} \tag{3}$$

The functions $f_{\xi k}(t)$ and $f_{\zeta k}(t)$ denote fault signals.

Remark 1: The variables ξ_i and ζ_i can be interpreted as position and velocity, respectively, for a mobile system, or as phase and frequency in the context of power networks.

Let $n = 2N$ and consider $x(t) \in \mathbb{R}^n$,the global state of the network, defined as $x(t) = [\xi_1(t), \ \ldots, \ \xi_N(t), \ \zeta_1(t), \ \ldots, \ \zeta_N(t)]^\top$. The closed-loop dynamics of the network in the presence of faults can be written as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bv(t) + B_f f(t) \\ y(t) &= Cx(t), \end{aligned} \tag{4}$$

where $v(t) \in \mathbb{R}^r$ is a vector of known external control inputs, $f(t) \in \mathbb{R}^m$ is a vector of fault signals, $y(t) \in \mathbb{R}^p$ is the output vector, and A, B, B_f, and C are matrices of appropriate dimensions. Before stating the problem addressed in this work, we define what is meant by fault *detectability* and *isolability*, according to [13].

Definition 1: Given the system (4), a fault $f_k(t) \in \mathbb{R}$ is said to be detectable if

$$\left. \frac{\partial y}{\partial f_k} \right|_{f_k = 0} \not\equiv 0. \tag{5}$$

In general terms, this means that a detectable fault should produce a change in the output.

Lemma 1: Definition 1 is equivalent to say that the system's Rosenbrock matrix

$$\begin{bmatrix} sI - A & b_{f_k} \\ C & 0 \end{bmatrix} \tag{6}$$

has full normal rank, where $b_{f_k} \in \mathbb{R}^m$ is the k-th column of B_f and normal rank is defined as the rank for almost all $s \in \mathbb{C}$.
This means that the transfer function from $f_k(t)$ to $y(t)$ is not identical to zero.

Definition 2: Given the system (4), a vector of faults $f(t) \in \mathbb{R}^m$ is said to be isolable if

$$\left. \frac{\partial y}{\partial f} \right|_{f=0} df \not\equiv 0. \tag{7}$$

In the case of additive faults, this relates to input observability and it loosely means that any simultaneous occurrence of faults would lead to a change in the output. Furthermore, the following can be said for additive faults

Lemma 2: Given the system (4), the m faults in $f(t)$ are isolable if and only if

$$\text{normal rank} \begin{bmatrix} sI - A & B_f \\ C & 0 \end{bmatrix} = n + m \tag{8}$$

In this paper, we solve the following problem:

Problem 1: When and how can each agent of the networked system (1)-(2) detect and isolate a faulty agent?
We propose a solution to this problem for two different consensus protocols (2) in the coming sections. However, first in the next section we introduce the mathematical tool to be used.

III. Preliminaries

A common technique used in model-based fault diagnosis is to generate a set of residuals which indicate the presence of a fault. The residual is a fault indicator computed from the difference between the measurements and their estimates, which should be close to zero if and only if the fault is not present. In this section, we consider the general linear fault-free system under the influence of an unknown input $d(t) \in \mathbb{R}^q$ to be described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bv(t) + Ed(t) \\ y(t) &= Cx(t) \end{aligned} \tag{9}$$

whereas the system in presence of faults is given by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bv(t) + B_f f(t) + Ed(t) \\ y(t) &= Cx(t) \end{aligned} \tag{10}$$

with the assumption that the matrices E and B_f have full column rank.

Remark 2: Note that the condition on B_f being full column rank is not restrictive, since any singular matrix $D \in \mathbb{R}^{n \times l}$ can be decomposed in $D = D_1 D_2$, with D_1 having full column rank. This implies, however, that not all faults are isolable.
The matrix E is called a disturbance distribution matrix, since it contains information on how a vector of unknown input signals, seen as disturbances, affect the states of the dynamical system.

Definition 3: A state observer is an unknown input observer (UIO) if the state estimation error $e = x - \hat{x}$ approaches zero asymptotically, regardless of the presence of an unknown input d.

A full-order observer for the fault-free system in (9) is described by:

$$\begin{aligned} \dot{z}(t) &= Fz(t) + TBv(t) + Ky(t) \\ \hat{x}(t) &= z(t) + Hy(t) \end{aligned} \tag{11}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the estimated state and $z(t) \in \mathbb{R}^n$ is the observer's state. Note that if we choose $F = A$, $T = I$, and $H = 0$ we have a full-order Luenberger observer. The matrices in the observer's equations must be designed in order to achieve the decoupling from the unknown input and meet requirements on the stability of the observer. Choosing the matrices F, T, K, H to satisfy the following conditions

$$\begin{aligned} F &= (A - HCA - K_1 C) \\ T &= (I - HC) \\ (HC - I)E &= 0 \\ K_2 &= FH \end{aligned} \tag{12}$$

we have the estimation error dynamics

$$\dot{e}(t) = Fe(t). \tag{13}$$

We conclude that if the equations in(12) are satisfied and F is stable, then the observer (11) is an UIO. The following proposition formalizes this.

Proposition 1 ([11]): The observer (11) is an UIO for (9) if and only if

i) $\text{rank}(CE) = \text{rank}(E)$
ii) (C, A_1) is a detectable pair, where

$$A_1 = A - HCA \tag{14}$$

For a proof and more details the reader is refered to [11], [13].

Now consider the system (10). As suggested in [11], a possible method of detecting and isolating the faults present in the process is to use the so called generalized observer scheme (GOS), where we construct a bank of observers generating a structured set of residuals such that each residual is decoupled from one and only one fault, but being sensitive to all other faults.

Definition 4: A residual $r(t)$ is a fault indicator function, which satisfies the following condition:

$$r(t) = 0 \iff f(t) = 0$$

where $f(t)$ represents the fault signal.

The detection and isolation of a fault in the k-th component,$f_k \neq 0$, is based on that:

$$\begin{aligned} \|r_k(t)\| &< \Theta_{f_k} \\ \|r_j(t)\| &\geq \Theta_{f_j} \ , \forall j \neq k, \end{aligned} \tag{15}$$

where $r_i(t)$ is the residual insensitive to a fault in the i-th component and $\Theta_{f_i} > 0$ is the isolation threshold, which can be constant or time varying. Note that this approach is feasible only if a single additive fault is present. If more faults are present, they can be detected using this method, but they cannot be isolated. To isolate multiple faults, one could enlarge the observer bank with multi-fault detectors.

Suppose there is a single active fault, $f_i(t) \neq 0$. In order to render an observer insensitive to $f_i(t)$, this fault could be regarded as an unknown input and the observer could then be computed using the UIO theory. The system (10) can be rewritten as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bv(t) + B_f^i f^i(t) + b_{f_i} f_i(t) \\ y(t) &= Cx(t) \end{aligned} \tag{16}$$

where b_{f_i} is the i-th column of B_f, $f_i(t)$ the i-th component of $f(t)$, B_f^i is B_f with the i-th column deleted and $f^i(t)$ the fault vector $f(t)$ with its i-th component removed. The UIO decoupled from b_{f_i} has the same structure as in (11) and is described by

$$\begin{aligned} \dot{z}_i(t) &= F_i z_i(t) + T_i Bv(t) + K_i y(t) \\ \hat{x}_i(t) &= z_i(t) + H_i y(t) \end{aligned} . \tag{17}$$

Remark 3: Note that for such a UIO to exist, $f_i(t)$ must be detectable. This follows from that Cond. ii) of Prop. 1 is equivalent to requiring the asymptotic stability of the transmission zeros of the system (A, b_{f_i}, C), which implies that

$$\text{normal rank} \begin{bmatrix} sI - A & b_{f_i} \\ C & 0 \end{bmatrix} = n + 1 \tag{18}$$

It is easy to show that we have the following observer error and residual dynamics

$$\begin{aligned} \dot{e}_i(t) &= F_i e_i(t) - T_i B_f^i f^i(t) \\ r_i(t) &= Ce_i(t) \end{aligned} . \tag{19}$$

Note that the residual dynamics are driven by the k-th fault if $T_i b_{f_k} \neq 0, k \neq i$. We can compute similar observers for all the other faults and then use the threshold logic in (15) to isolate the fault.

From Remark 3 we conclude that the existence of UIOs for all additive faults requires such faults to be detectable. Together with the assumption that B_f has full-column rank, we conclude that the existence of a bank of UIOs ensures the isolability of all additive faults.

Next we show that one can construct such UIO for two consensus protocols applied to a networked system.

IV. Fault detection for networked systems

Consider the networked system introduced in Section II with the following consensus protocol

$$m_i u_i(t) = -d_i \zeta_i(t) + \sum_{j \in N_i} w_{ij} \left(\xi_j(t) - \xi_i(t) \right). \tag{20}$$

where $m_i, w_{ij}, d_i > 0 \in \mathbb{R}$ and $\xi_i \in \mathbb{R}$. Recall the networked system (4), with $x(t) = [\xi_1(t), \cdots, \xi_N(t), \zeta_1(t), \cdots, \zeta_N(t)]^\top$ and

$$\begin{aligned} A &= \begin{bmatrix} 0_N & I_N \\ -\bar{M}\mathcal{L} & -\bar{D}\bar{M} \end{bmatrix} \\ B &= \begin{bmatrix} 0_N \\ \bar{M} \end{bmatrix} \\ \bar{M} &= \text{diag}\left(\frac{1}{m_1}, \cdots, \frac{1}{m_N} \right) \\ \bar{D} &= \text{diag}\left(d_1, \cdots, d_N \right). \end{aligned} \tag{21}$$

Assume that ξ_k does not satisfy Equation (1a), but

$$\dot{\xi}_k(t) = \zeta_k(t) + f_k(t) \tag{22}$$

where $f_k(t)$ corresponds to a fault in node k. In the presence of this fault, (4) transforms into

$$\dot{x}(t) = Ax(t) + b_f^k f_k(t) \tag{23}$$

with $b_f^k = [\bar{b}_f^{k\top}\ 0_{1\times N}]^\top$ where $\bar{b}_f^k$ is an N dimensional vector with all zero entries except one that corresponds to the faulty node k. Furthermore, we assume node i has access to

$$y_i(t) = C_i x(t), \quad C_i = \left[\bar{C}_i\ 0_{\tilde{N}_i \times N}\right], \tag{24}$$

with $\bar{C}_i$ being an $|\tilde{N}_i|$ by N matrix with full row rank, where each of the rows have all zero entries except for one entry at the j-th position that corresponds to those nodes that are neighbors of i, where $\tilde{N}_i = N_i \cup i$ and $j \in \tilde{N}_i$.

To tackle Problem 1 we need to show that one can construct a UIO at each node i under the consensus protocol (20) using measurements (24).

Before presenting the main result of this paper we have the following lemma.

Lemma 3: If an undirected graph $\mathcal{G}$ is connected, then any partition of its Laplacian matrix $\mathcal{L}$, induced by a strict subset of nodes $\bar{\mathcal{V}} \subset \mathcal{V}$, is invertible.

Proof: See [17]. ■

Now we are ready to state the following theorem concerning the existence of a UIO for the consensus protocol (20).

Theorem 1: There exists a UIO for the system (23) with measurements (24) of node i if the graph $\mathcal{G}$ is connected and $k \in N_i$.

Proof: First we have to show that

$$\text{rank}\left(C_i b_f^k\right) = \text{rank}\left(b_f^k\right) = 1.$$

Denote the row of C_i that reads the output of node k, c_i^k. It is obvious that $c_i^k b_f^k = 1$ and $c_i^j b_f^k = 0,\ j \neq k$. Hence, $C_i b_f^k$ is a vector with zero entries except one which is equal to 1, thus the rank is equal to 1. Then we have to show that

$$\text{rank}(\mathcal{D}) = 2N + 1$$

for all $\mathrm{Re(s)} \geq 0$ where

$$\mathcal{D} = \begin{bmatrix} sI_{2N} - A & b_f^k \\ C_i & 0_{\tilde{N}_i \times 1} \end{bmatrix}.$$

We have

$$\text{rank}(\mathcal{D}) = \text{rank}\begin{bmatrix} sI_N & -I_N & \bar{b}_f^k \\ \bar{M}\mathcal{L} & sI_N + \bar{D}\bar{M} & 0_{N\times 1} \\ \bar{C}_i & 0_{|\tilde{N}_i|\times N} & 0_{|\tilde{N}_i|\times 1} \end{bmatrix}$$

Applying some row and column operations we obtain

$$\text{rank}(\mathcal{D}) = \text{rank}\begin{bmatrix} 0_N & -I_N & \bar{b}_f^k \\ a(s) & 0_N & b(s) \\ \bar{C}_i & 0_{|\tilde{N}_i|\times N} & 0_{|\tilde{N}_i|\times 1} \end{bmatrix},$$

with

$$\begin{aligned} a(s) &= s^2 I_N + s\bar{D}\bar{M} + \bar{M}\mathcal{L} \\ b(s) &= (sI_N + \bar{D}\bar{M})\bar{b}_f^k \end{aligned}.$$

We apply a transformation P to the system so that

$$\begin{aligned} \bar{x} = Px = [&\xi_{\tilde{i}_1},\ \cdots,\ \xi_{\tilde{i}_{|\tilde{N}_i|}},\ \xi_{\bar{i}_1},\ \cdots,\ \xi_{\bar{i}_{|\bar{N}_i|}}, \\ &\zeta_{\tilde{i}_1},\ \cdots,\ \zeta_{\tilde{i}_{|\tilde{N}_i|}},\ \zeta_{\bar{i}_1},\ \cdots,\ \zeta_{\bar{i}_{|\bar{N}_i|}}]^\top, \end{aligned}$$

where $\tilde{i}_j \in \tilde{N}_i$, $\bar{i}_j \in \bar{N}_i$, and $\bar{C}_i^* = \bar{C}_i P = [I_{\tilde{N}_i} 0_{\tilde{N}_i \times \bar{N}_i}]$, where $\tilde{N}_i = i \cup N_i$ and $\bar{N}_i = \mathcal{V} \setminus \tilde{N}_i$. After this operation we can write the Laplacian as $\bar{\mathcal{L}} = P^{-1}\mathcal{L}P = \begin{bmatrix} \mathcal{L}_{|\tilde{N}_i|} & l_{|\tilde{N}_i|\times|\bar{N}_i|} \\ l_{|\bar{N}_i|\times|\tilde{N}_i|} & \mathcal{L}_{|\bar{N}_i|} \end{bmatrix}$. Furthermore $P^{-1}\bar{M}P = \begin{bmatrix} \bar{M}_{1|\tilde{N}_i|} & 0_{|\tilde{N}_i|\times|\bar{N}_i|} \\ 0_{|\bar{N}_i|\times|\tilde{N}_i|} & \bar{M}_{2|\bar{N}_i|} \end{bmatrix}$, $P^{-1}\bar{D}P = \begin{bmatrix} \bar{D}_{1|\tilde{N}_i|} & 0_{|\tilde{N}_i|\times|\bar{N}_i|} \\ 0_{|\bar{N}_i|\times|\tilde{N}_i|} & \bar{D}_{2|\bar{N}_i|} \end{bmatrix}$, $\tilde{b}_f^k = P^{-1}\bar{b}_f^k$, and $\tilde{b}_f^{k*} = P^{-1}(sI_N + \bar{D}\bar{M})\bar{b}_f^k$.

After applying the transformation we have

$$\text{rank}(\mathcal{D}) = \\ \text{rank}\begin{bmatrix} 0_{|\tilde{N}|\times|\tilde{N}_i|} & 0_{|\tilde{N}_i|\times|\bar{N}_i|} & -I_N & \tilde{b}_f^k \\ c(s) & \bar{M}_1 l_{|\tilde{N}_i|\times|\bar{N}_i|} & 0_{|\tilde{N}_i|\times N} & \tilde{b}_f^{k*} \\ \bar{M}_2 l_{|\bar{N}_i|\times|\tilde{N}_i|} & d(s) & 0_{|\bar{N}_i|\times N} & 0_{|\bar{N}_i|\times 1} \\ I_{|\tilde{N}_i|} & 0_{|\tilde{N}_i|\times|\bar{N}_i|} & 0_{|\tilde{N}_i|\times N} & 0_{|\tilde{N}_i|\times 1} \end{bmatrix},$$

with

$$\begin{aligned} c(s) &= \bar{M}_1\mathcal{L}_{|\tilde{N}_i|} + s^2 I_{|\tilde{N}_i|} + s\bar{M}_1\bar{D}_1 \\ d(s) &= \bar{M}_2\mathcal{L}_{|\bar{N}_i|} + s^2 I_{|\bar{N}_i|} + s\bar{M}_2\bar{D}_2 \end{aligned}.$$

It is evident that the first and the third columns are independent of the rest, thus

$$\begin{aligned} \text{rank}(\mathcal{D}) &= |\tilde{N}_i| + N + \\ &+ \text{rank}\begin{bmatrix} \bar{M}_1 l_{|\tilde{N}_i|\times|\bar{N}_i|} & \tilde{b}_f^{k*} \\ \bar{M}_2\mathcal{L}_{|\bar{N}_i|} + s^2 I_{|\bar{N}_i|} + s\bar{M}_2\bar{D}_2 & 0_{|\bar{N}_i|\times 1} \end{bmatrix}. \end{aligned}$$

We know from Lemma 3 that any partition of the Laplacian matrix is invertible so the last column is independent of the rest as well so

$$\text{rank}(\mathcal{D}) = |\tilde{N}_i| + N + |\bar{N}_i| + 1 = 2N + 1 \tag{25}$$

■

Remark 4: Note that if the graph is not connected, the networked system (23) can be decomposed in several decoupled subsystems, each corresponding to a connected subset of the network. Theorem 1 then applies to each subsystem.

Theorem 1 establishes that a UIO can be constructed at node i that can observe node k. The existence of such observer leads to detection of a possible fault at node k by node i using the method described in Section III.

In Theorem 1 we stated that a fault in ξ_k can be isolated with the measurements of the form (24). In the next theorem we identify what type of faults cannot be isolated.

Theorem 2: Consider the system described by (23). For any of the following pairs of C_i and b_f^k no UIO of the form (11) exists:

i) $b_f^k = [\bar{b}_f^{k\top}\ 0_{1\times N}]^\top$, $C_i = \left[0_{\tilde{N}_i\times N}\ \bar{C}_i\right]$
ii) $b_f^k = [0_{1\times N}\ \bar{b}_f^{k\top}]^\top$, $C_i = \left[0_{\tilde{N}_i\times N}\ \bar{C}_i\right]$
iii) $b_f^k = [0_{1\times N}\ \bar{b}_f^{k\top}]^\top$, $C_i = \left[\bar{C}_i\ 0_{\tilde{N}_i\times N}\right]$

Proof: To see that no UIO exists for (i) and (iii) one needs to check that

$$\text{rank}\left(C_i b_f^k\right) = \text{rank}\left(b_f^k\right) = 0.$$

Hence, the first condition of Proposition 1 is not satisfied. For (ii), similar to the calculations in proof of Theorem 1 for the case where $s=0$, we have

$$\operatorname{rank}(\mathcal{D}) = \operatorname{rank}\begin{bmatrix} 0_N & -I_N & \bar{b}_f^k \\ \bar{M}\mathcal{L} & 0_N & \bar{D}\bar{M}\bar{b}_f^k \\ 0_{\tilde{N}_i\times N} & \bar{C}_i & 0_{|\tilde{N}_i|\times 1} \end{bmatrix}. \tag{26}$$

Considering the first column, it is known that $\mathcal{L}$ is rank deficient, and hence the second condition of Theorem 1 is not satisfied. ■

Cases i) and iii) from Theorem 2 suggest that if there is an unknown input affecting one of the states of one of the nodes in a network, it is not possible to have a UIO without measuring the same state throughout the network as the one affected by the unknown input. For example, if an unknown input(fault) is affecting the velocity of one of the nodes, by measuring positions alone we cannot have a UIO to observe the states of the network. On the other hand, in case ii) we see that the first condition of Proposition 1 is satisfied, but the UIO still does not exist. What happens in this case is that the system is not detectable, as seen by observing the first two columns of (26).

In what comes next we introduce the conditions where a UIO exists for observing ζ_j $j\in N_i$ and consequently detecting a fault in them.

Theorem 3: Consider the system described by (23)–(24). For $C_i = \begin{bmatrix} \bar{C}_i & 0_{|\tilde{N}_i|\times N} \\ 0_{|\tilde{N}_i|\times N} & \bar{C}_i \end{bmatrix}$, where $\bar{C}_i$ is a $|\tilde{N}_i|$ by N matrix, and $b_f^{k\top} = \begin{bmatrix} 0_{1\times N} & \bar{b}_f^{k\top} \end{bmatrix}$ and $\bar{b}_f^k$ being an N by 1 vector with having k-th entry as its only nonzero entry, a UIO exists to observe ζ_j $j\in N_i$

Proof: The proof for existence of a UIO is similar to the previous case and is omitted ■

For the rest of this section we consider another consensus protocol [18]:

$$u_i(t) = \sum_{j\in N_i} w_{ij}\left[(\xi_j(t)-\xi_i(t)) + \gamma(\zeta_j(t)-\zeta_i(t))\right], \tag{27}$$

Furthermore, for the whole network with a faulty node k and the same selection of x we have

$$\dot{x}(t) = Ax(t) + b_f^k f_k(t) \tag{28}$$

where

$$A = \begin{bmatrix} 0_N & I_N \\ -\mathcal{L} & -\gamma\mathcal{L} \end{bmatrix}, \tag{29}$$

and $\mathcal{L}$ is the weighted Laplacian matrix with the weight $w_{ij} > 0$, $\gamma>0$, $b_f^{k\top} = \begin{bmatrix} \bar{b}_f^{k\top} & 0_{1\times N} \end{bmatrix}$ with $\bar{b}_f^k$ being an N by 1 vector with having k-th entry as its only nonzero entry. We further assume that node i measures $y_i(t)$ at time t which satisfies

$$y(t) = C_i x(t), \tag{30}$$

and $C_i = \begin{bmatrix} \bar{C}_i & 0_{|\tilde{N}_i|\times N} \\ 0_{|\tilde{N}_i|\times N} & \bar{C}_i \end{bmatrix}$, where $\bar{C}_i$ is a $|\tilde{N}_i|$ by N matrix. Now we have the following theorem.

Theorem 4: There exists a UIO for the system (28) if the graph $\mathcal{G}$ is connected, measurements of the form (30) are available and the faulty node k is in the neighborhood of node i, N_i.

Proof: The proof is very similar to that of Theorem 1 and is omitted. ■

For detecting fault in $\zeta_k(t)$, $j\in N_k$ we have the following theorem.

Theorem 5: For $b_f^{k\top} = \begin{bmatrix} 0_{1\times N} & \bar{b}_f^{k\top} \end{bmatrix}$ with $\bar{b}_f^k$ being an N by 1 vector with having k-th entry as its only nonzero entry, a UIO exists to observe $\zeta_k(t)$, $k\in N_i$.

Proof: The proof for existence of a UIO is similar to the previous case and is omitted ■

So far we have established what type of measurements should be available at node i to be able to detect a fault in $k\in N_i$ using a UIO based fault detection scheme. More specifically we have shown that if a node aims to detect a fault in a state of one of its neighbors using the aforementioned UIO based scheme, it has to measure the same state of all of its neighbors.

V. Power Networks Application

Power systems are an example of very complex systems in which generators and loads are dynamically interconnected. Thus they can be seen as networked systems, where each bus is a node. We will now provide a simple model for the active power flow in a power grid. Such model and additional details of power networks can be found in [19].

The behavior of a bus i can be described by the so-called *swing equation*:

$$m_i\ddot{\delta}_i(t) + d_i\dot{\delta}_i(t) - P_{mi}(t) = -\sum_{j\in N_i} P_{ij}(t), \tag{31}$$

where m_i and d_i are the inertia and damping coefficients, respectively, P_{mi} is the mechanical input power and P_{ij} is the active power flow from bus i to j. Considering that there are no power losses nor ground admittances and letting $V_i = |V_i|\, e^{j\delta_i}$ and δ_i be, respectively, the complex voltage and the phase angle of bus i, the active power flow between bus i and bus j, P_{ij}, is given by:

$$P_{ij}(t) = k_{ij}\sin(\delta_i(t)-\delta_j(t)), \tag{32}$$

where $k_{ij} = |V_i|\,|V_j|\, b_{ij}$ and b_{ij} is the susceptance of the power line connecting buses i and j.

Since the phase angles are close, we can linearize (32), rewriting the dynamics of bus i as:

$$m_i\ddot{\delta}_i(t) + d_i\dot{\delta}_i(t) = u_i(t) + v_i(t), \tag{33}$$

with

$$\begin{aligned} u_i(t) &= -\sum_{j\in N_i} k_{ij}(\delta_i(t)-\delta_j(t)) \\ v_i(t) &= P_{mi}. \end{aligned} \tag{34}$$

Consider a power network with $\mathcal{G}(\mathcal{V},\mathcal{E})$ as its underlying graph with $N = |\mathcal{V}|$ nodes, where each node corresponds to a bus in the power network. Rewriting (33) and (34) in state-state form and considering $x =$

$\left[\delta_1(t), \; \cdots, \; \delta_N(t), \dot{\delta}_1(t), \; \cdots, \; \dot{\delta}_N(t)\right]^\top$, we can write the network's dynamics as

$$\dot{x}(t) = Ax(t) + Bv(t), \tag{35}$$

where $B = \left[0_N \; \bar{M}^\top\right]^\top$, A and $\bar{M}$ are given by (21) and $v(t) = [P_{m1} \; \cdots \; P_{mN}]^\top$ is the collection of input power at each bus. These are generator's power inputs or load power consumptions, which we assume as known. The dynamics of the power network correspond to (21) with an additional known input $v(t)$ and thus the results from Section IV can be used to detect and isolate faults in power networks.

Remark 5: The stability and convergence properties of the system $\dot{x}(t) = Ax(t)$ where $d_1 = \cdots = d_N$ are studied in [20], and the case where d_i, $i = 1, \cdots, N$ are not necessarily equal is not presented here due to lack of space.

In the example that follows next, we consider that the network is being affected by faults corresponding to unexpected changes in the power generation or consumption. Assume that a fault has occurred at node k. The power network under such conditions can be modeled as

$$\dot{x} = Ax + Bv(t) + b_f^k f_k, \tag{36}$$

where b_f^k is the k-th column of B and therefore it can be written as $b_f^k = \left[0_{1\times N} \; \bar{b}_f^{k\top}\right]^\top$ with $\bar{b}_f^{k\top}$ being a column vector with $\frac{1}{m_k}$ in the k-th entry and zero in all other entries. Thus, from Theorem 3 there exists an UIO for such system at a given node i if $k \in N_i$ and $y_i = C_i x$ with

$$C_i = \begin{bmatrix} \bar{C}_i & 0_{|\tilde{N}_i|\times N} \\ 0_{|\tilde{N}_i|\times N} & \bar{C}_i \end{bmatrix}.$$

Thus we need to measure the phase and frequency of the neighbors to be able to detect the faulty node. Having such measurements, this type of faults can be detected and isolated in a distributed way using UIOs, as we show with the following example.

Consider the power network presented in Fig. 1. The power grid's topological parameters and the generators' dynamic coeficients (m_i and d_i) were taken from [21], while the dynamic coeficients of the rest of the buses were arbitrarily taken from reasonable values. The system matrices used in the simulation can be found in the appendix.

The power network is evolving towards the steady-state when, at time instant $t = 2s$, a fault occurs at node 6, as presented in Fig. 2. By implementing a bank of observers at bus 7, the fault is successfully detected and isolated in the presence of measurement noise, since the residual corresponding to bus 6 became larger than the other residuals, as illustrated in Fig. 3.

Remark 6: Because of Theorem 2 we know that we cannot solve the fault detection problem using UIO with having access to less information than the information available through $y_i = C_i x$, with the above-mentioned C_i.

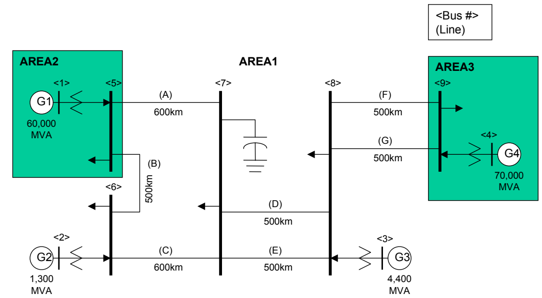

Figure 1. Power network with 9 buses [21].

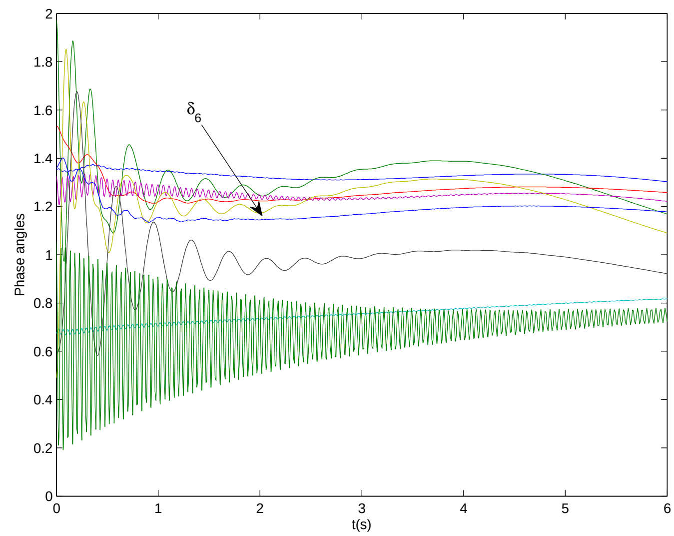

Figure 2. Phase angles of the power network.

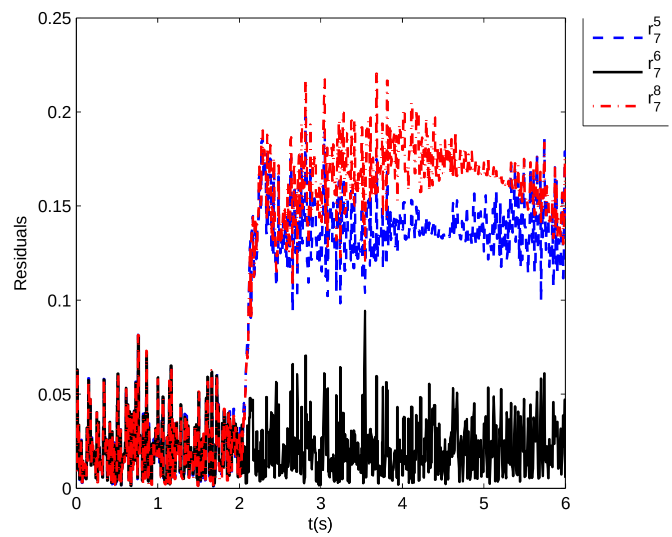

Figure 3. Residuals of buses neighboring bus 7.

VI. Concluding Remarks and Future Directions

In this paper we considered the problem of fault detection and isolation for interconnected nodes with double-integrator dynamics performing consensus. We presented an illustrative example to show the application of the proposed method to fault detection in power systems. Future directions include considering a way to reduce the dimension of the unknown input observers at each node in the current scheme, and explore applicability of other fault detection methods to the problem.

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Appendix

$$A = \begin{bmatrix} A_1 & A_2 \\ I_9 & 0_9 \end{bmatrix}$$

$$B = \begin{bmatrix} B_1 \\ 0_9 \end{bmatrix}$$

$$C = \begin{bmatrix} \bar{C} & 0_9 \\ 0_9 & \bar{C} \end{bmatrix}$$

$$\bar{C} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} -25.8844 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -22.7101 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -15.1515 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -15.2672 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.5003 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.5752 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.4452 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.4532 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.6322 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -697.98 & 0 & 0 & 0 & 697.98 & 0 & 0 & 0 & 0 \\ 0 & -873.11 & 0 & 0 & 0 & 873.11 & 0 & 0 & 0 \\ 0 & 0 & -507.47 & 0 & 0 & 0 & 0 & 507.47 & 0 \\ 0 & 0 & 0 & -488.74 & 0 & 0 & 0 & 0 & 488.74 \\ 12016.97 & 0 & 0 & 0 & -12230.33 & 113.25 & 100.11 & 00 & \\ 0 & 486.52 & 0 & 0 & 148.43 & -763.75 & 128.80 & 0 & 0 \\ 0 & 0 & 0 & 0 & 102.69 & 100.81 & -274.10 & 70.60 & 0 \\ 0 & 0 & 1135.52 & 0 & 0 & 0 & 71.40 & -1276.58 & 69.66 \\ 0 & 0 & 0 & 18635.24 & 0 & 0 & 0 & 75.18 & -18710.42 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0.1355 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5.4881 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0818 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0685 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.3334 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3.0581 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2.3935 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.4207 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.6126 \end{bmatrix}$$