Security Metrics for Control Systems

from Analysis to Design

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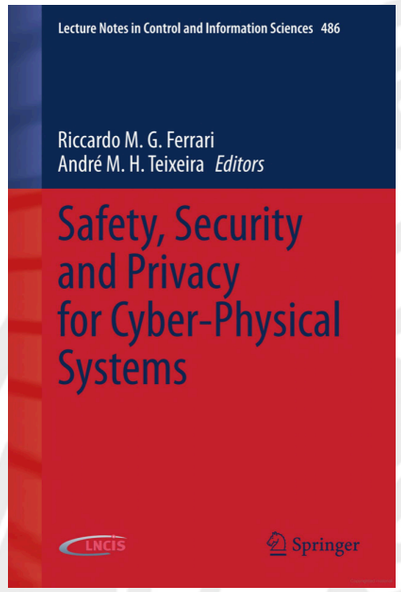
Outline

- Cybersecurity in Control Systems
  - Security Game
  - Risk Management
    - Mapping to Control System Design
    - Example: undetectable attacks

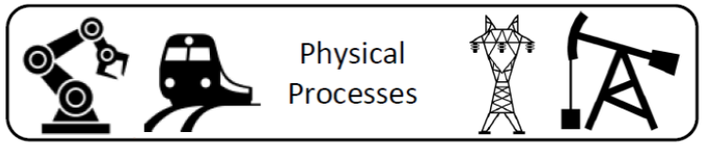
- Security Metrics for Control Systems
  - Classical metrics in control engineering
  - Novel security metric for analysis and design
    - Analysis and design problems
    - Structural limitations - invariant zeros

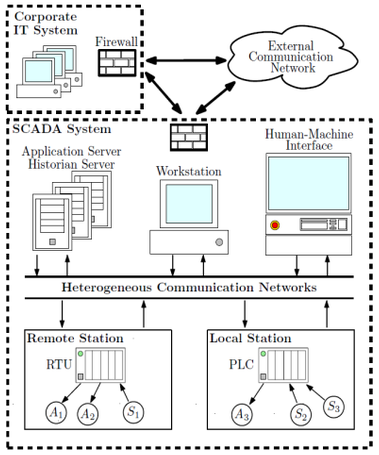
- Additional topics
  - Variations of the adversary models and their respective metrics
  - Incorporating uncertainty for robust open-loop attacks

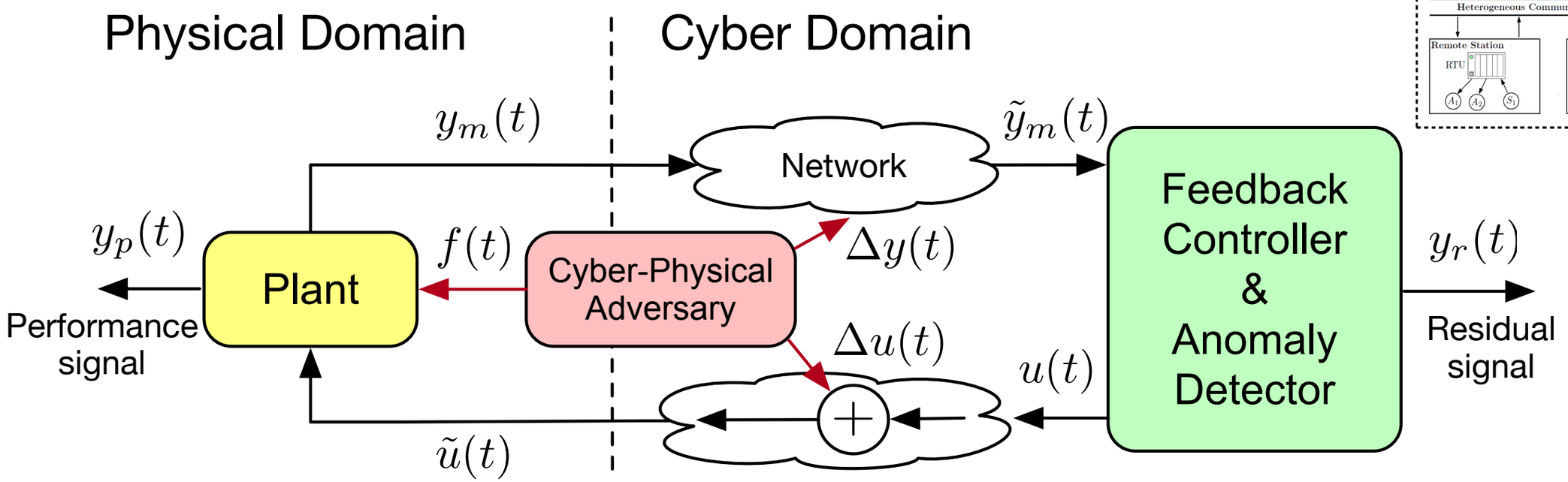


A. M. H. Teixeira, “Security metrics for control systems,” in Safety, Security and Privacy for Cyber-Physical Systems, R. M. Ferrari and A. M. H. Teixeira, Eds. Cham: Springer International Publishing, 2021.

“The Security Game”: key ingredients







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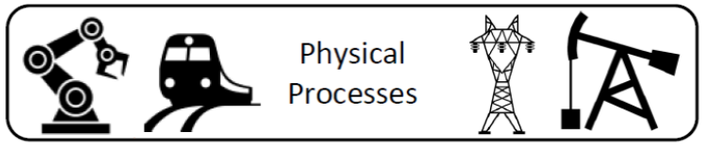

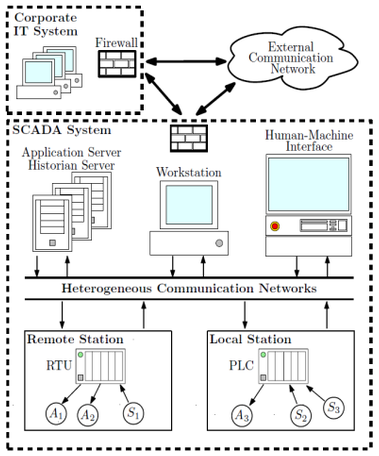

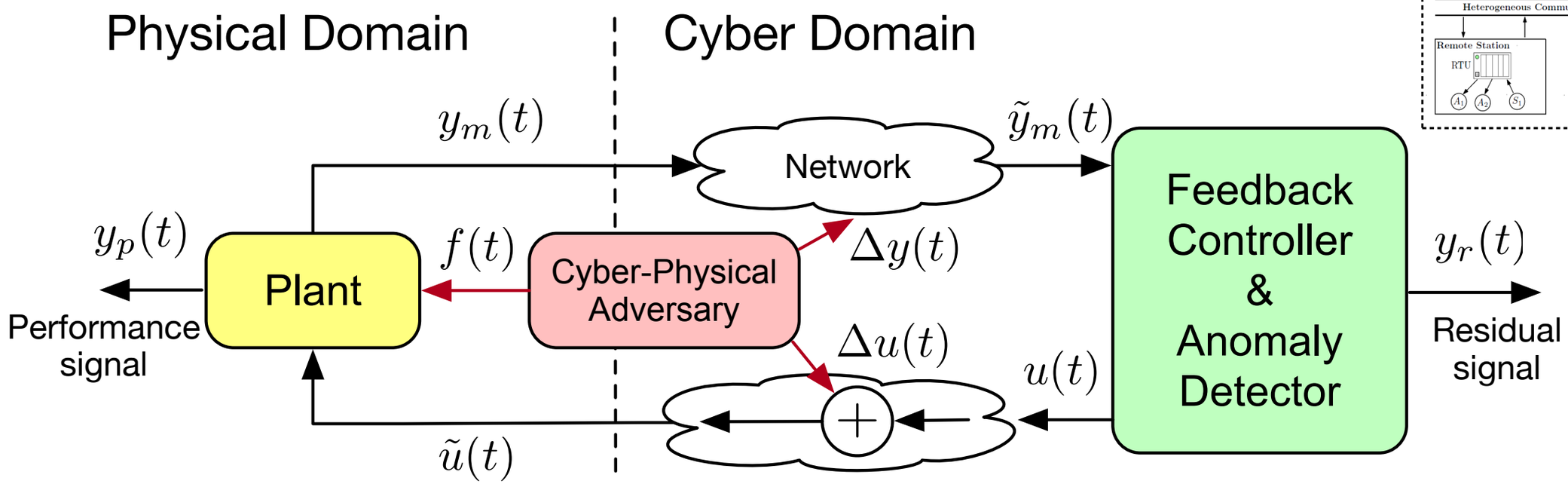

Need tools and strategies to understand and mitigate attacks:

- **Which threats** should we care about?
- **What impact** can we expect from attacks?
- **Which resources** should we **protect**, and how?

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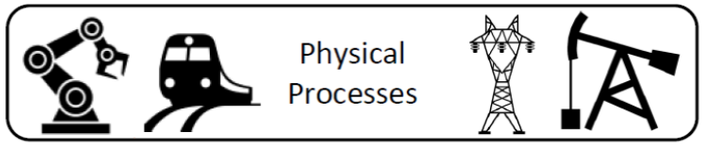

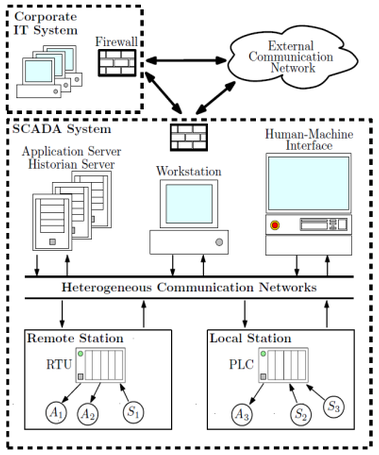

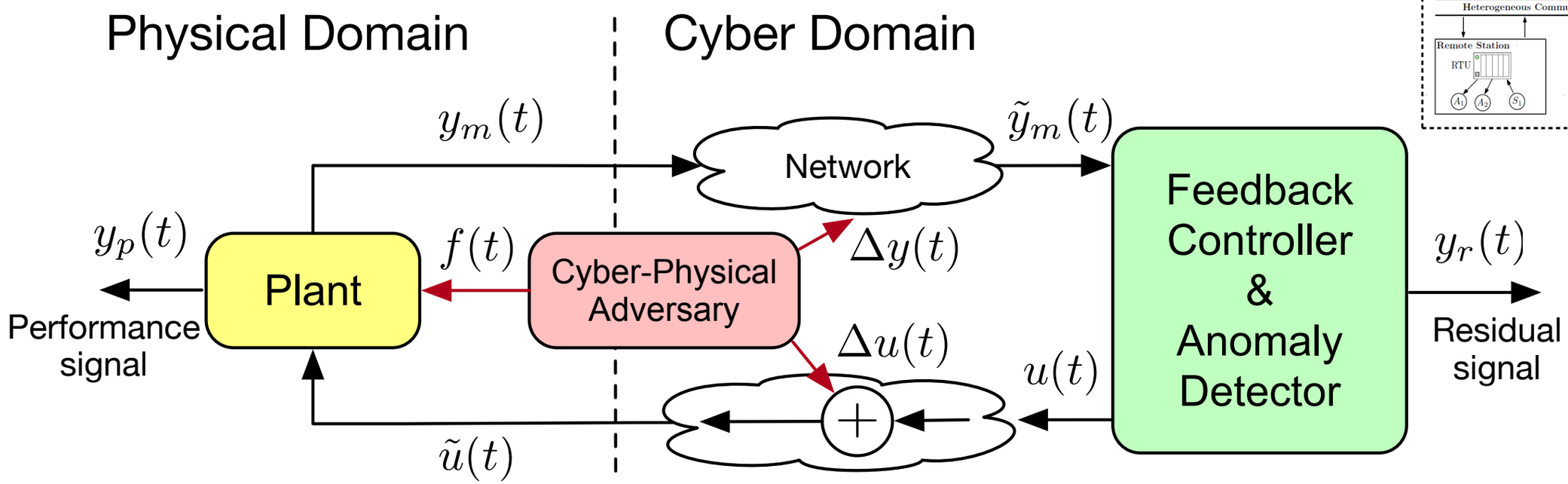

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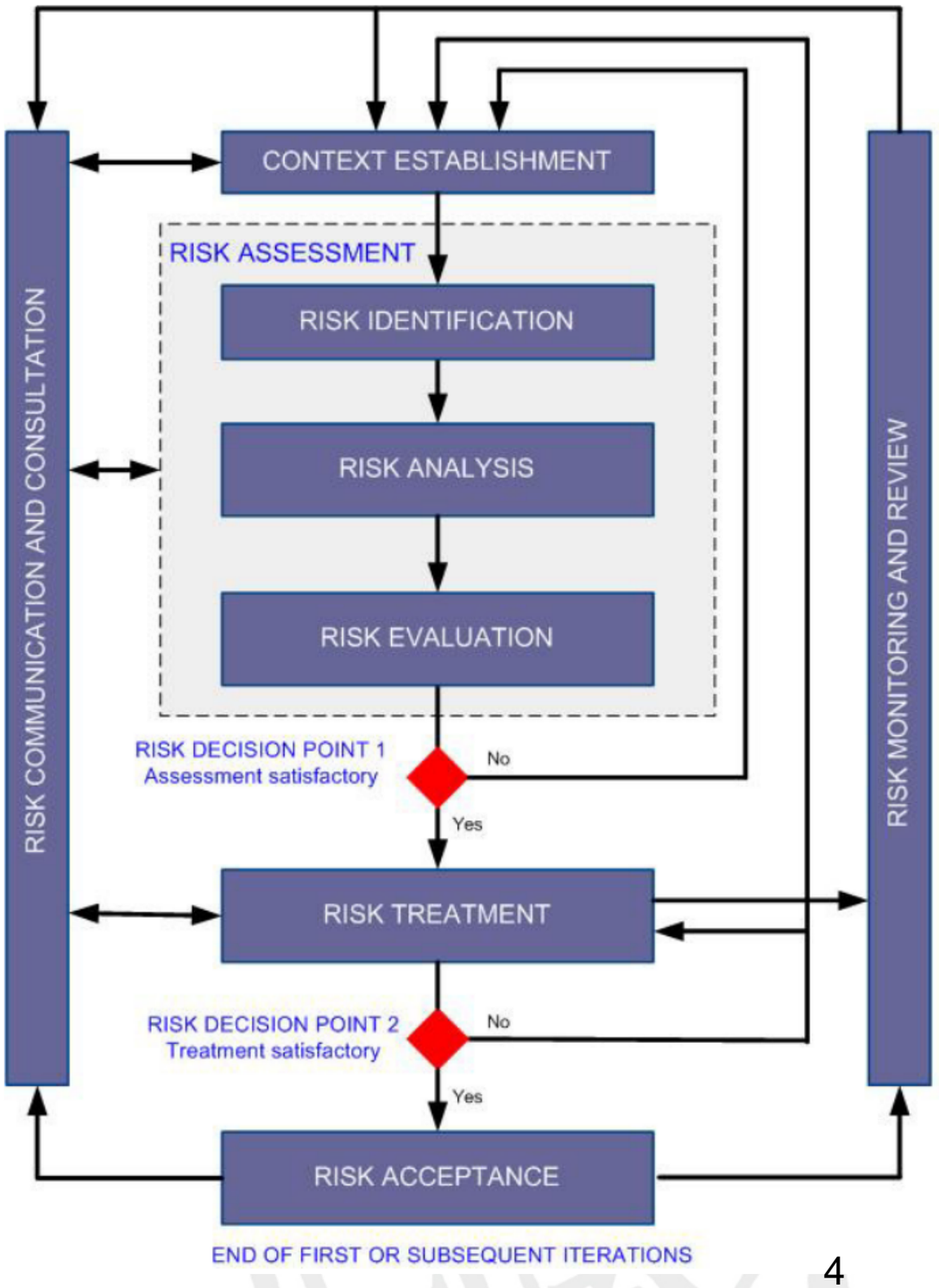
- How to find answers: **Risk Management + Control Engineering**

Risk Management Cycle

Risk = (Scenario, Likelihood, Impact)

[Kaplan & Garrick, 1981]



[ISO 31000]

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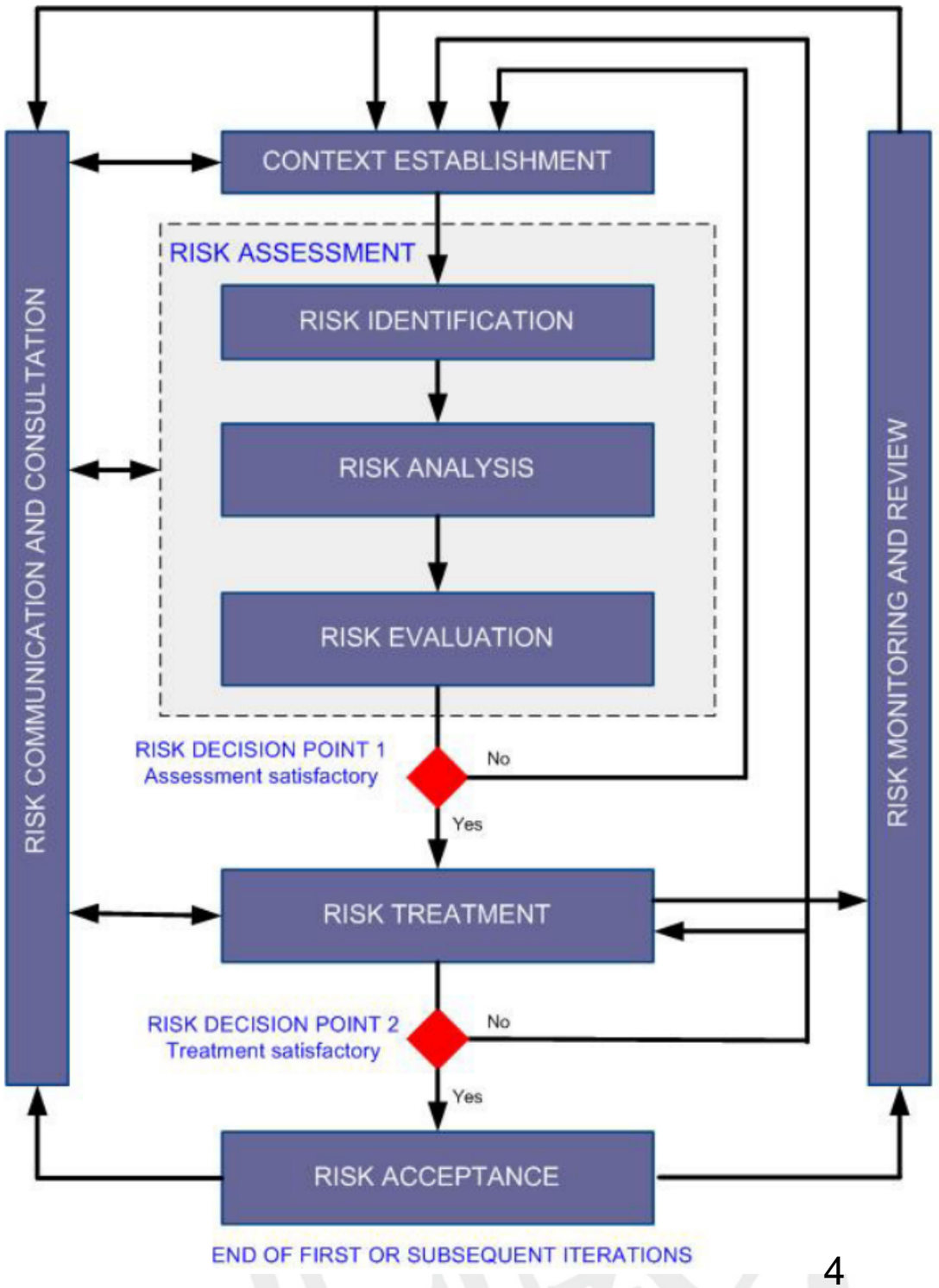
Main steps in risk management



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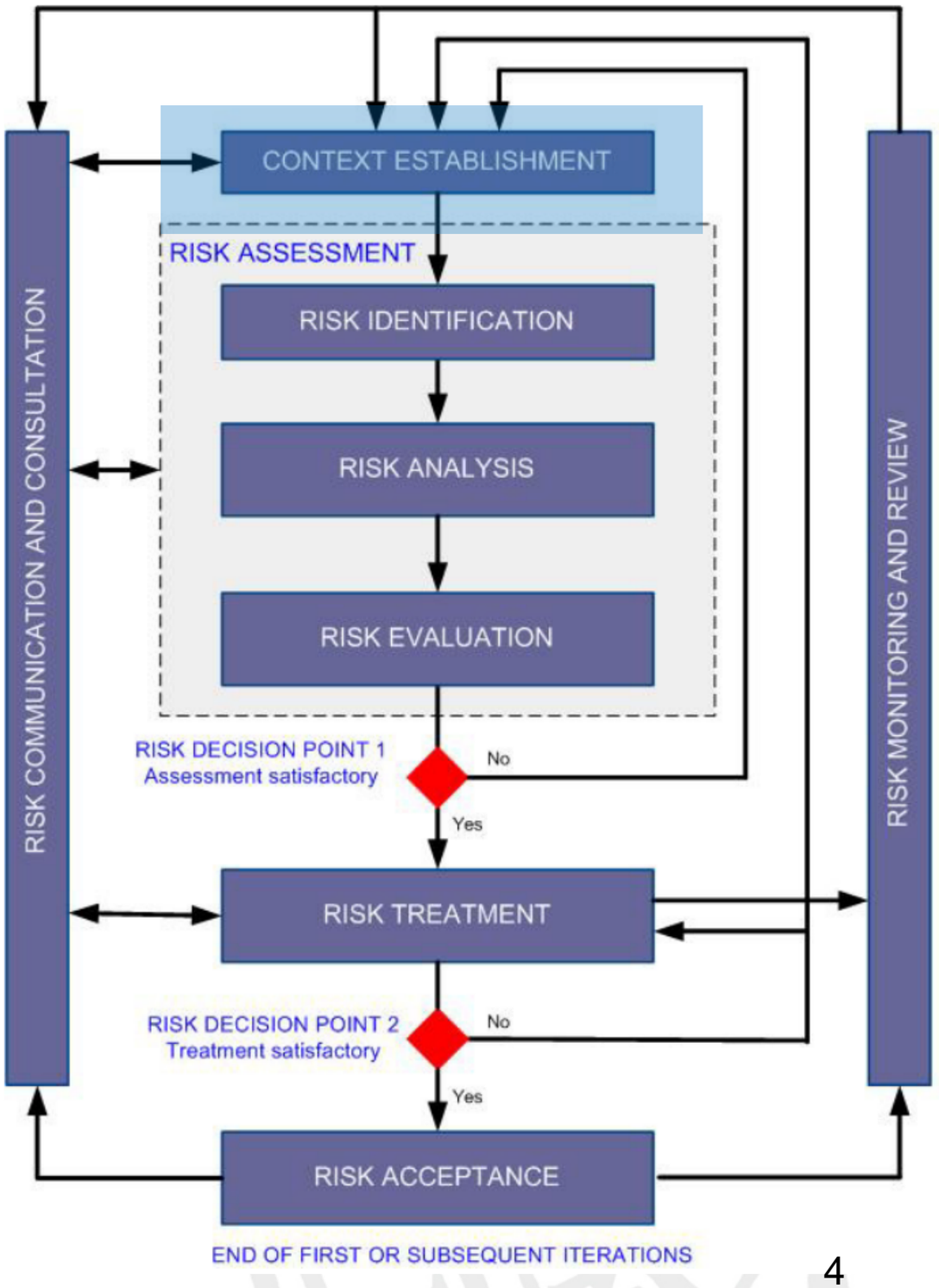

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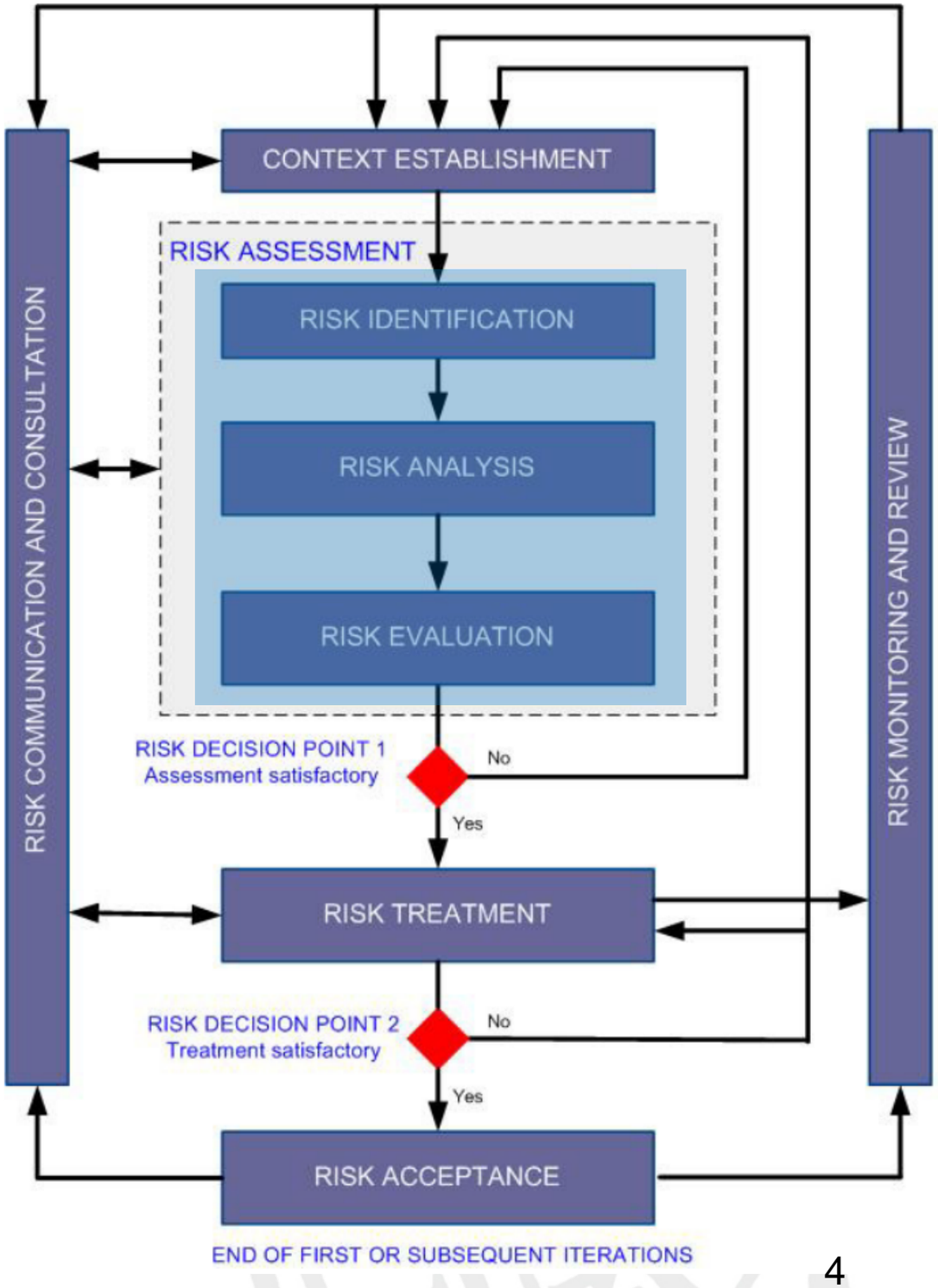

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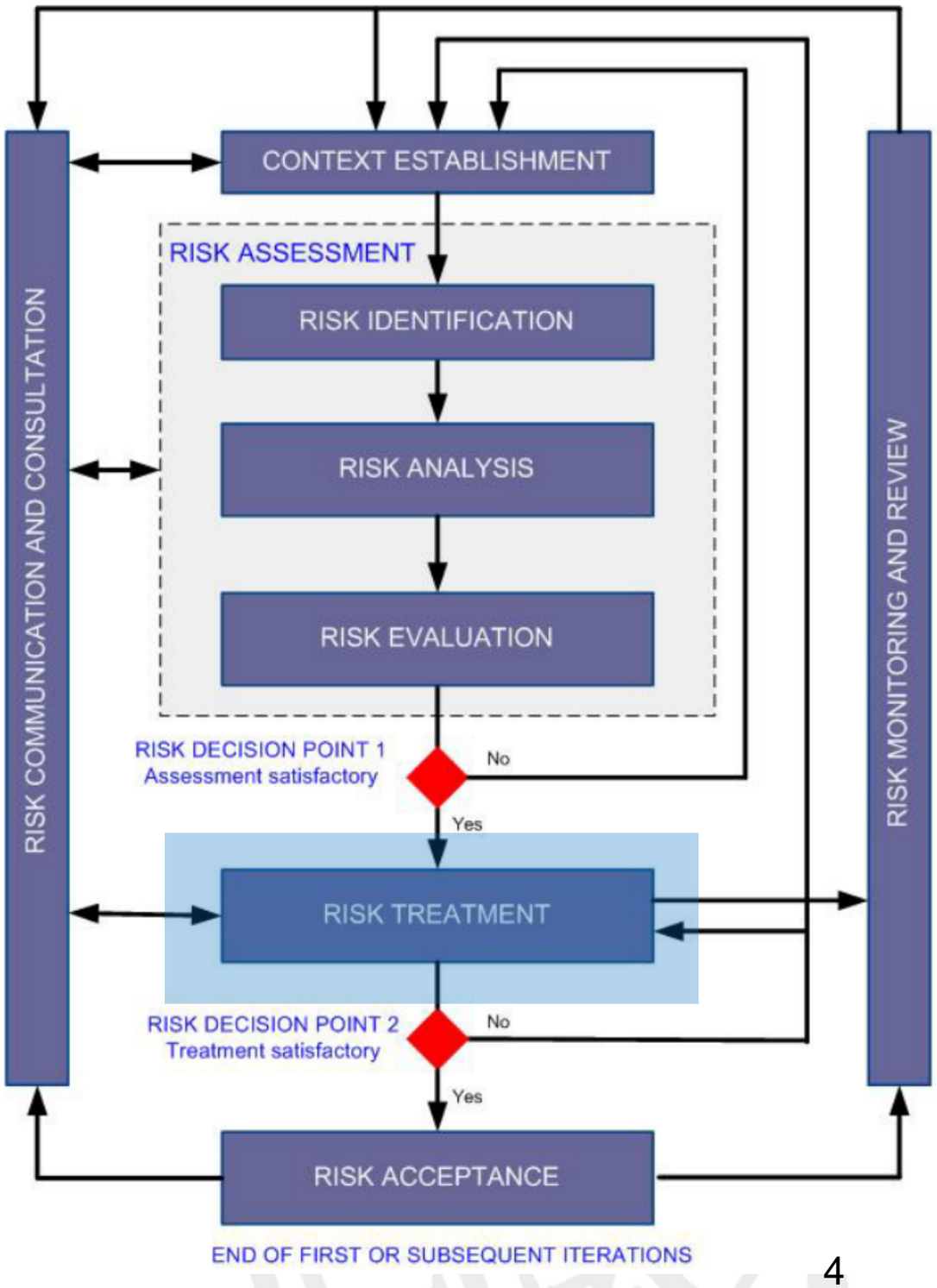
  - Likelihood Assessment
  - Impact Assessment
- Risk Mitigation
  - Prevention, Detection, Treatment



[ISO 31000]

Risk Management vs Control System Design

Main steps in risk management

- Scenario characterization
  - Models, Scenarios, Objectives

- Risk Analysis
  - Likelihood Assessment
  - Impact Assessment

- Risk Mitigation
  - Prevention, Detection, Treatment

Risk Management vs Control System Design

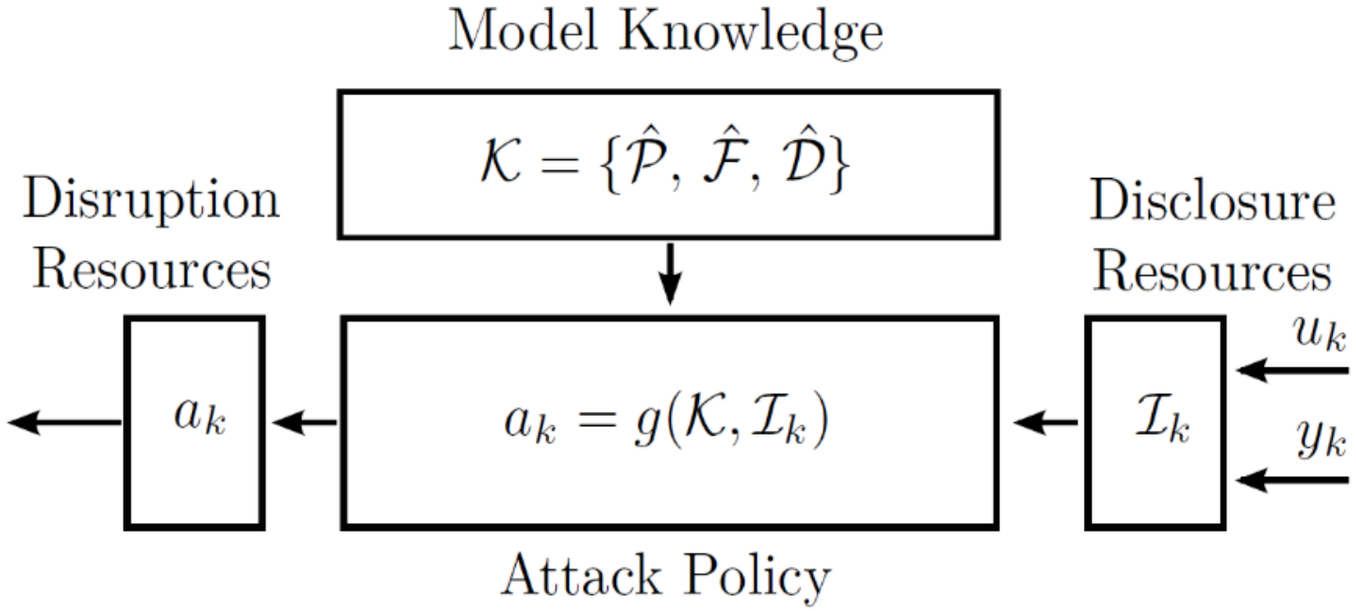
Main steps in risk management

- Scenario characterization
  - Models, Scenarios, Objectives
- Risk Analysis
  - Likelihood Assessment
  - Impact Assessment
- Risk Mitigation
  - Prevention, Detection, Treatment

Main steps in control system design:

- Scenario characterization
  - Models, Scenarios, Cost functions
- System Analysis
  - Structural: Controllability, observ. …
  - Performance: Robustness, optimality
- System Design
  - Structure of the system
  - Monitoring algorithms
  - Control algorithms

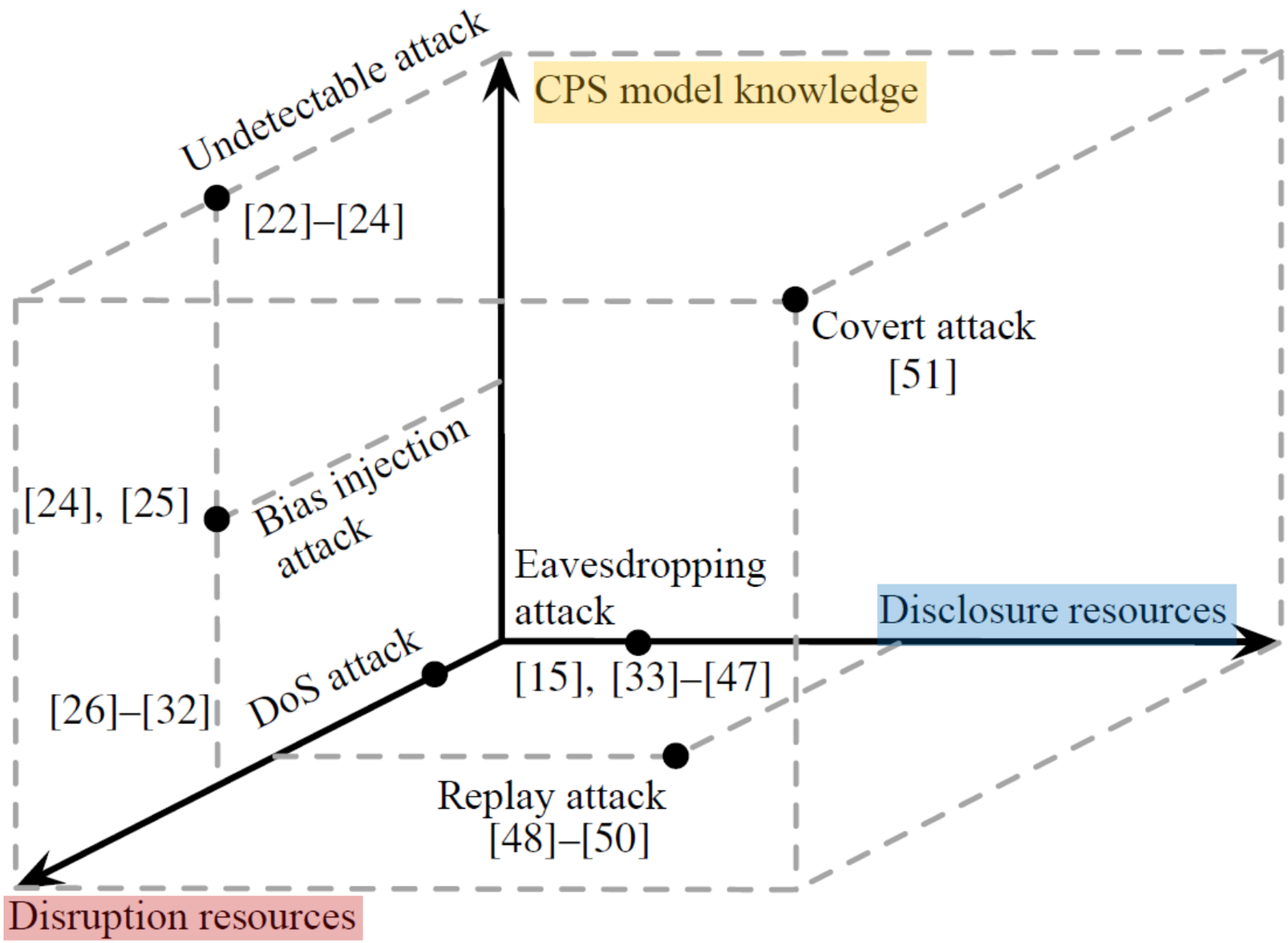
Overview - Adversary Models

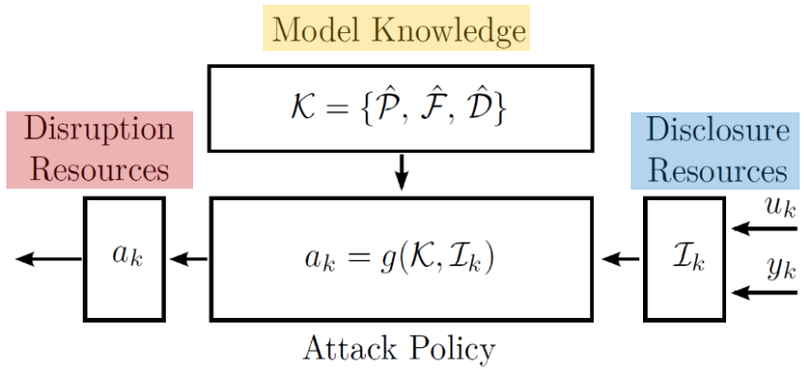


[Teixeira *et al*., Automatica, 2015]

- **Attack policy:** Goal of the attack? Destroy equipment, increase costs, *remain undetected*…
- **CPS model knowledge:** Adversary knows models of plant and controller? Better models increase possibility for stealthy attacks…
- **Disruption/disclosure resources:** Which channels can the adversary access?

Overview - Attack Scenarios





[Chong *et al.*, ECC, 2019] [Teixeira *et al.*, Automatica, 2015]

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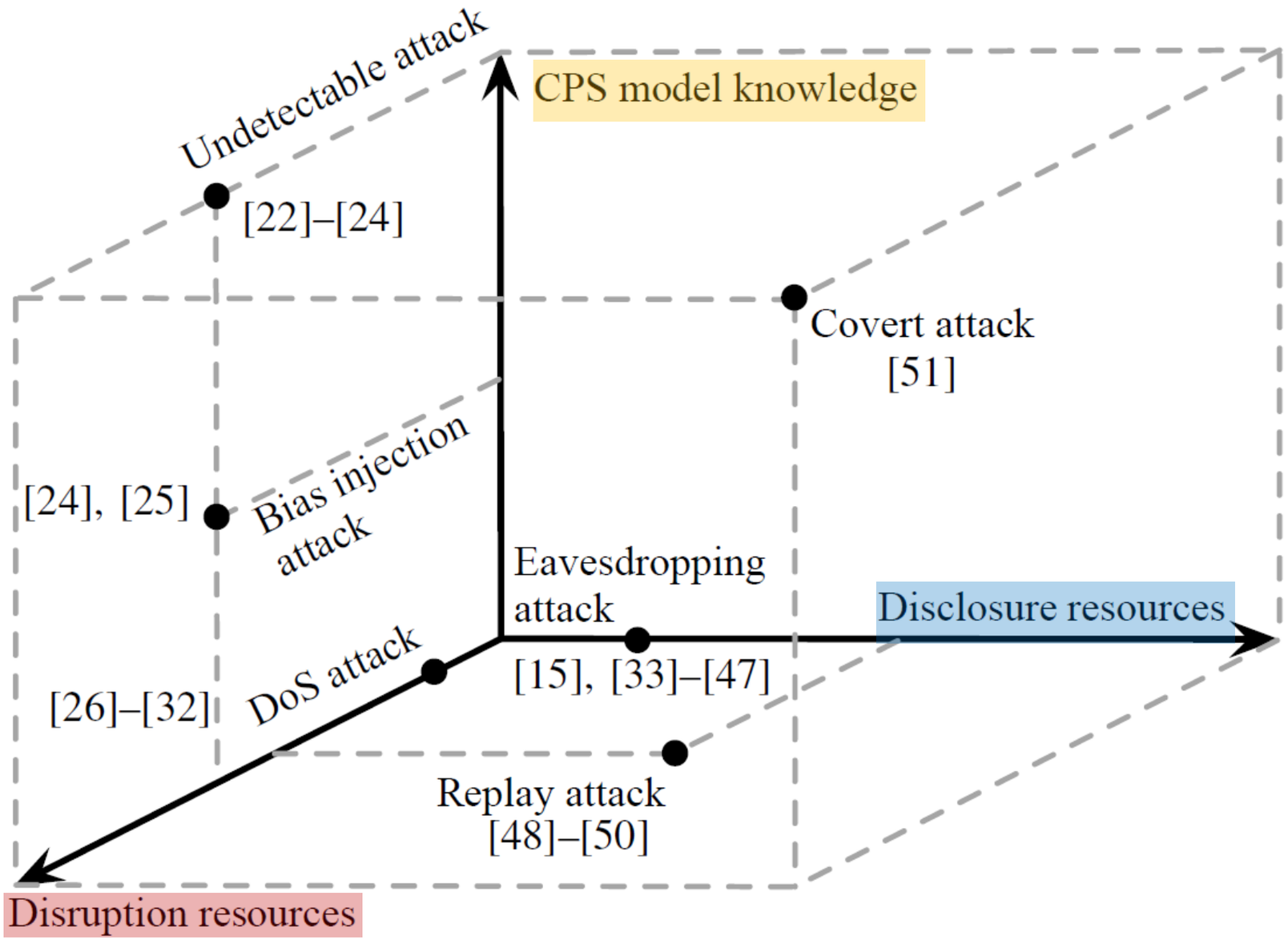
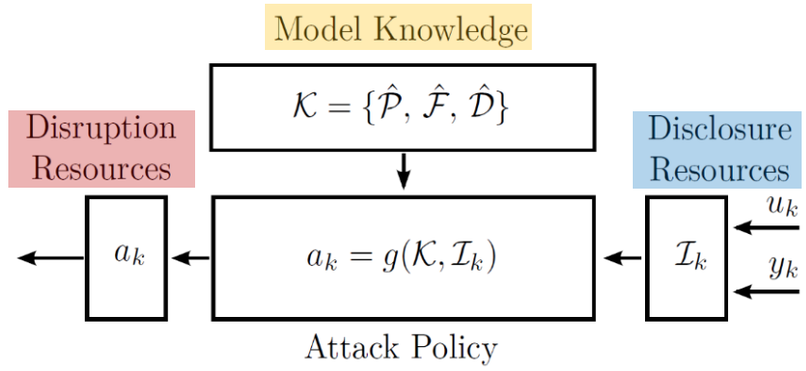
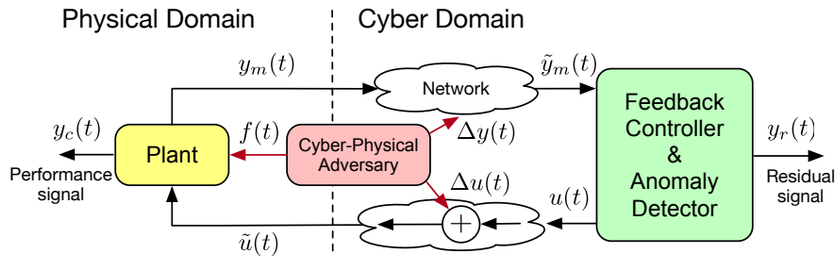




- **Adversary Models**
  - How does the adversary behave?
- **Security Analysis:**
  - Can it evade detection?
  - Can it violate safety?
- **Tailored Detection / Mitigation Schemes**

[Chong *et al*., ECC, 2019] [Teixeira *et al*., Automatica, 2015]

Undetectable Attacks



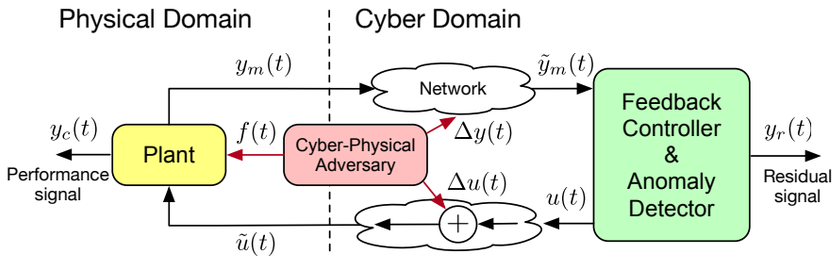
System dynamics:

$$x(k+1) = Ax(k) + Bu(k) + B_a a(k)$$

$$y(k) = Cx(k) + D_a a(k)$$

$$a_k = \begin{bmatrix} \Delta u_k \\ \Delta y_k \end{bmatrix}$$

Undetectable Attacks



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Output function: $y(k) = \Phi(x_0, a, k) \triangleq CA^k x_0 + C \sum_{i=1}^{k-1} A^{k-i-1} B_a a_i + D_a a_k$

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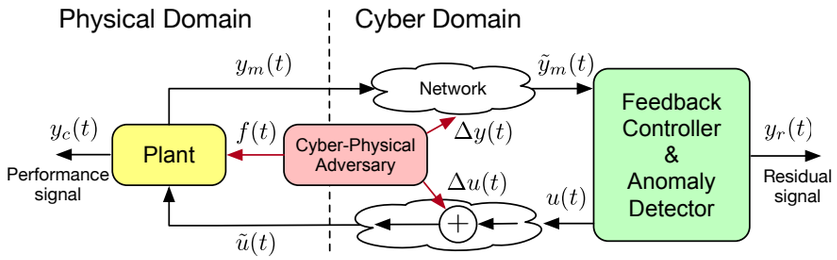

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Measurement trajectory under attack a: $y(k) = \Phi(x_0, a, k), \quad k \geq 0$

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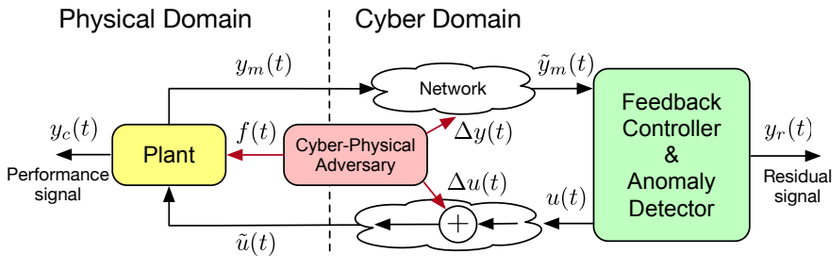

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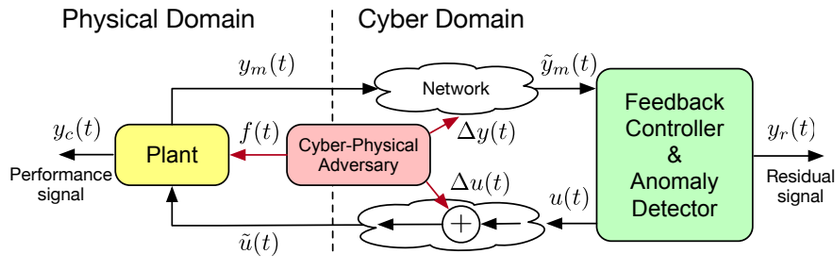
Definition: Attack a is *undetectable* if

$$\Phi(x_0, a, k) = \Phi(x_0^a, 0, k)$$

for *some* initial state x_0^a for all $k \geq 0$

Undetectable Attacks



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Interpretation: the output under attack can be confused as the result of an initial state without attack

Undetectable Attacks and the Zero Dynamics

Undetectability requires (for all $k \geq 0$)

$$0 = \Phi(x_0, a, k) - \Phi(x_0^a, 0, k)$$

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$$0 = \Phi(\bar{x}_0^a, a, k)$$

$\Leftrightarrow$ Must exist initial state $\bar{x}_0^a$ and input a_k yielding zero output.

This corresponds to the zero dynamics of the system: a well-known concept in control theory!

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Theorem: There exists undetectable attacks if and only if exist $\nu \in \mathbb{C}$ and $\bar{x}_0^a \in \mathbb{C}^{n_x}$, $g \in \mathbb{C}^{n_a}$ such that

$$\begin{bmatrix} \nu I_{n_x} - A & -B_a \\ C & D_a \end{bmatrix} \begin{bmatrix} \bar{x}_0^a \\ g \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

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In general requires system knowledge $\mathcal{K} = \{\mathcal{P}, \mathcal{F}, \emptyset\} = \{\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}\}$

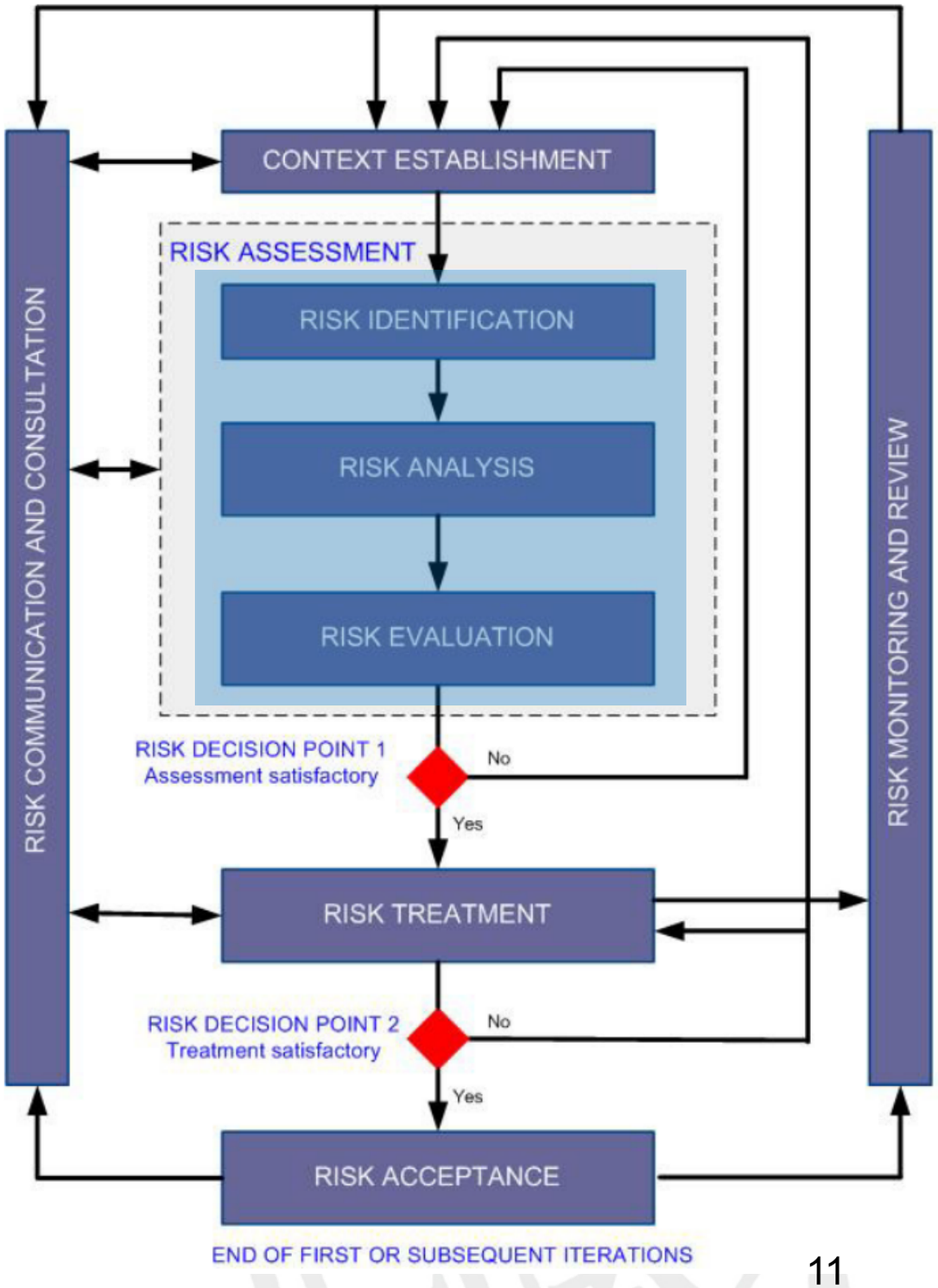
(Matlab function to compute ν: ν = tzero(ss(A, B_a, C, D_a))

Risk Management Cycle

Risk = (Scenario, Likelihood, Impact)
[Kaplan & Garrick, 1981]

Main steps in risk management

- Scenario characterization
  - Models, Scenarios, Objectives
- **Risk Analysis**
  - Likelihood Assessment
  - **Impact Assessment**
- Risk Mitigation
  - Prevention, Detection, Treatment



[ISO 31000]

Impact analysis

Attack Model:

Impact analysis

Attack Model:

- Zero dynamics:

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- Attack policy: $a_k = \nu^k g$
  - $|\nu| < 1$: vanishing attack
  - $|\nu| > 1$: diverging attack

Impact analysis

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Impact:

A vanishing attack will decay towards zero. It has a small impact.

Impact analysis

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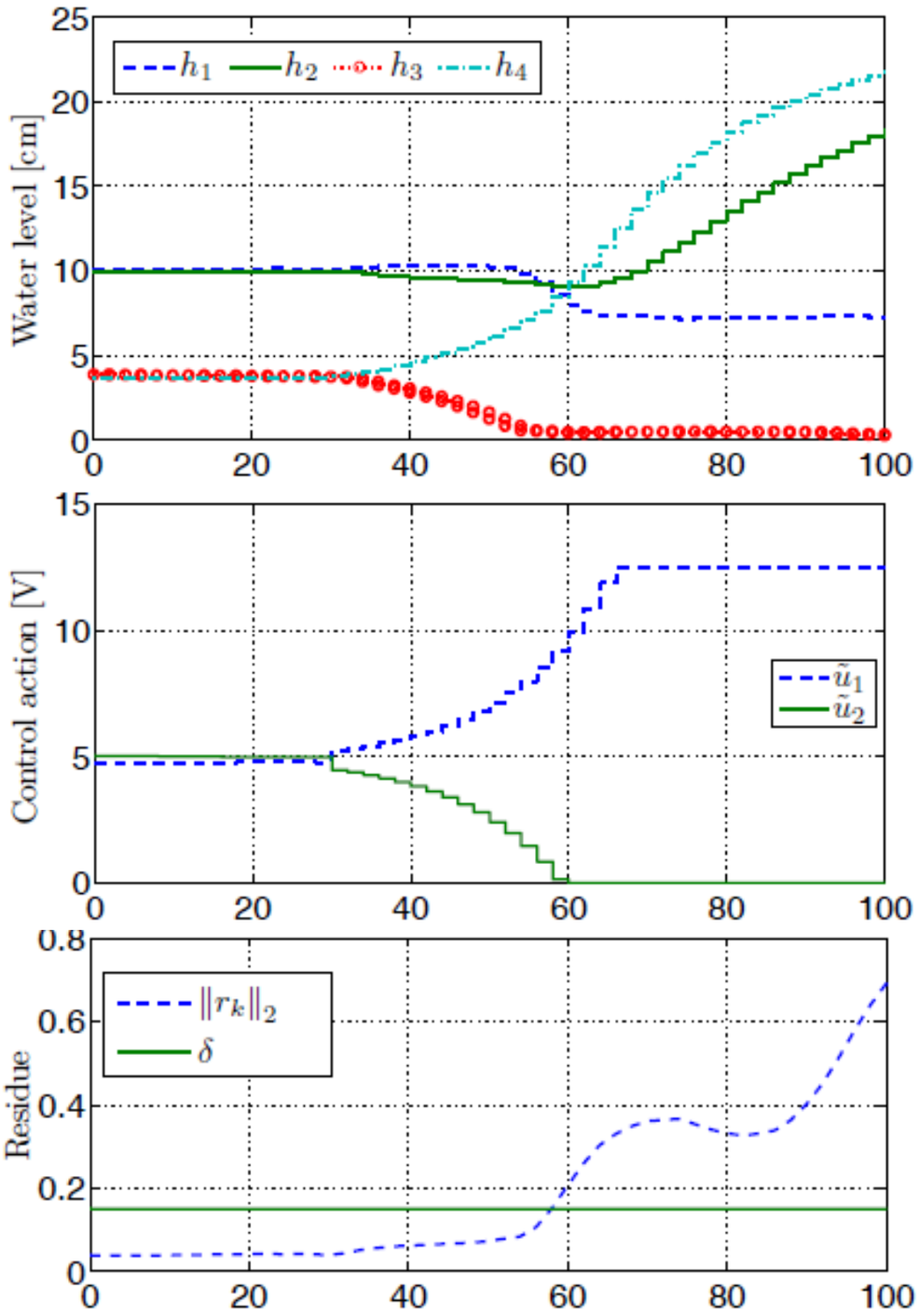
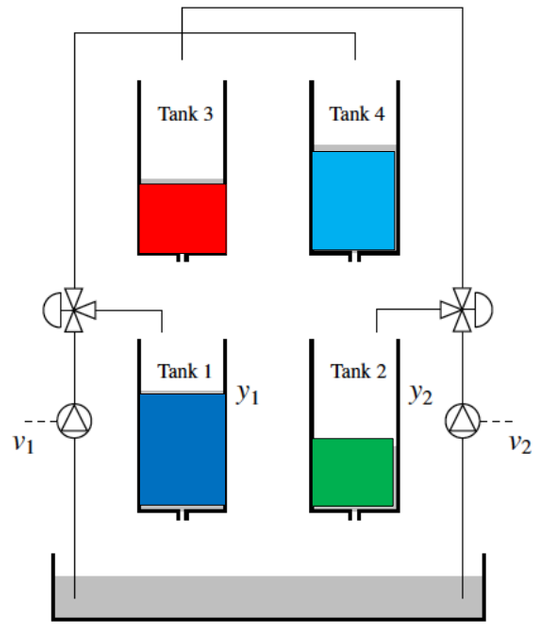
Impact:

A vanishing attack will decay towards zero. It has a small impact.

A diverging attack will increase towards infinity.
It has a large impact, if not detected.

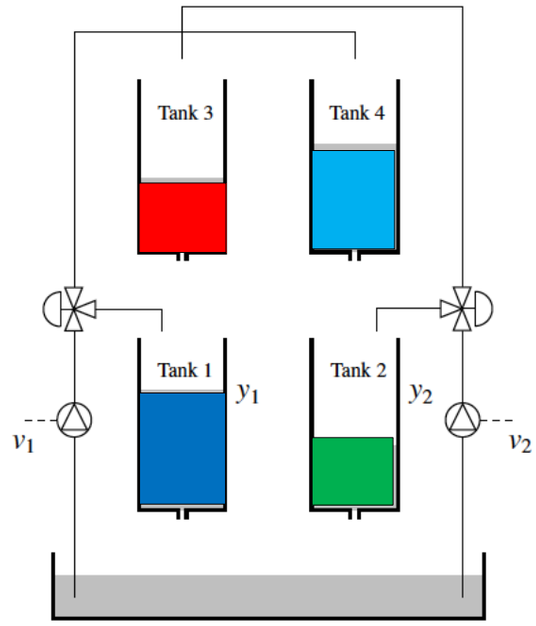
Zero Dynamics Attack - Experiment

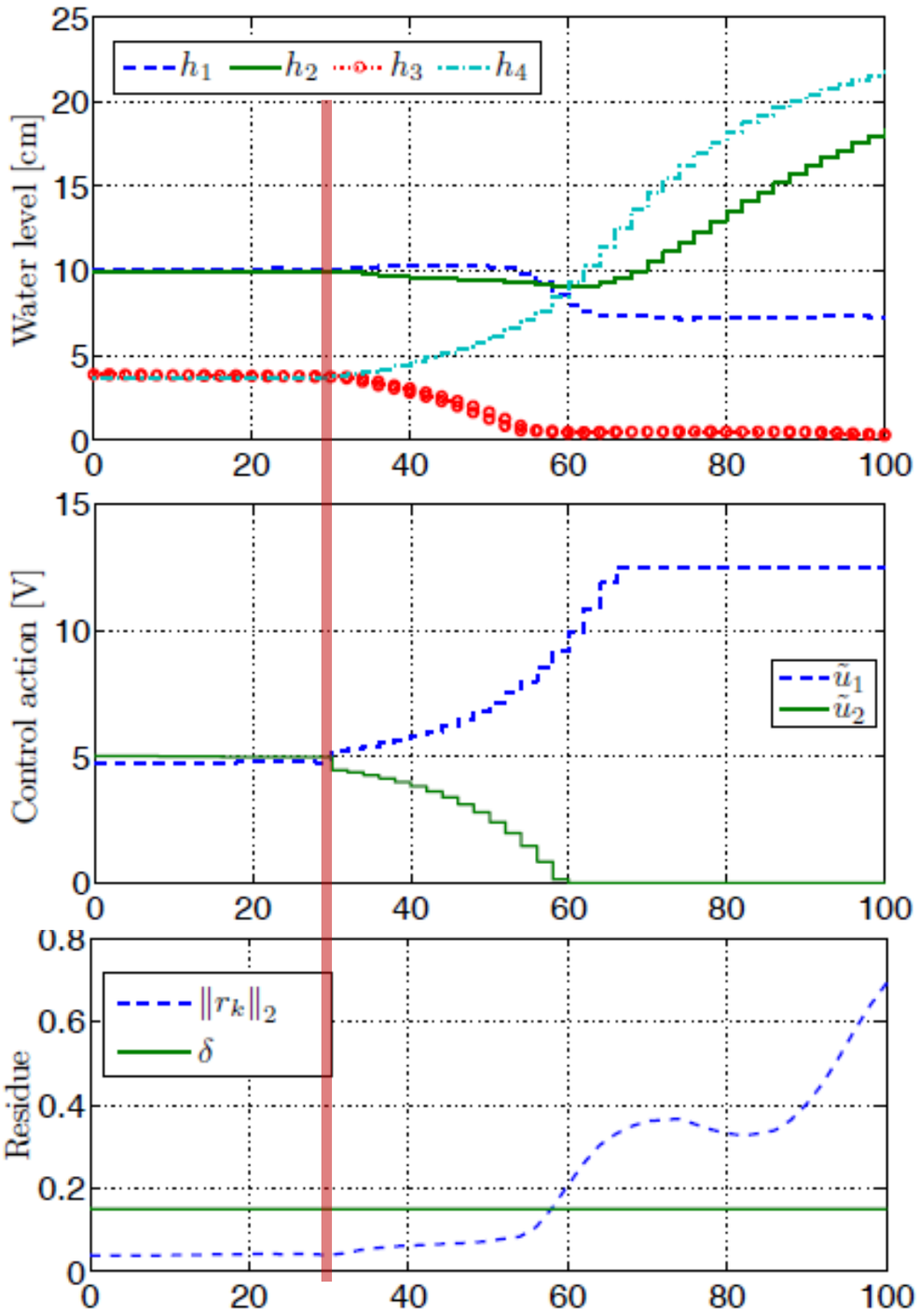




Attack Goal: Empty tank 3

Zero Dynamics Attack - Experiment





Attack Goal: Empty tank 3

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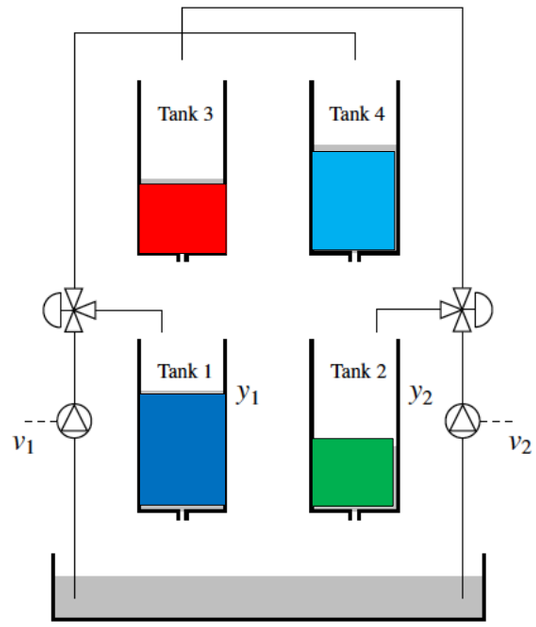

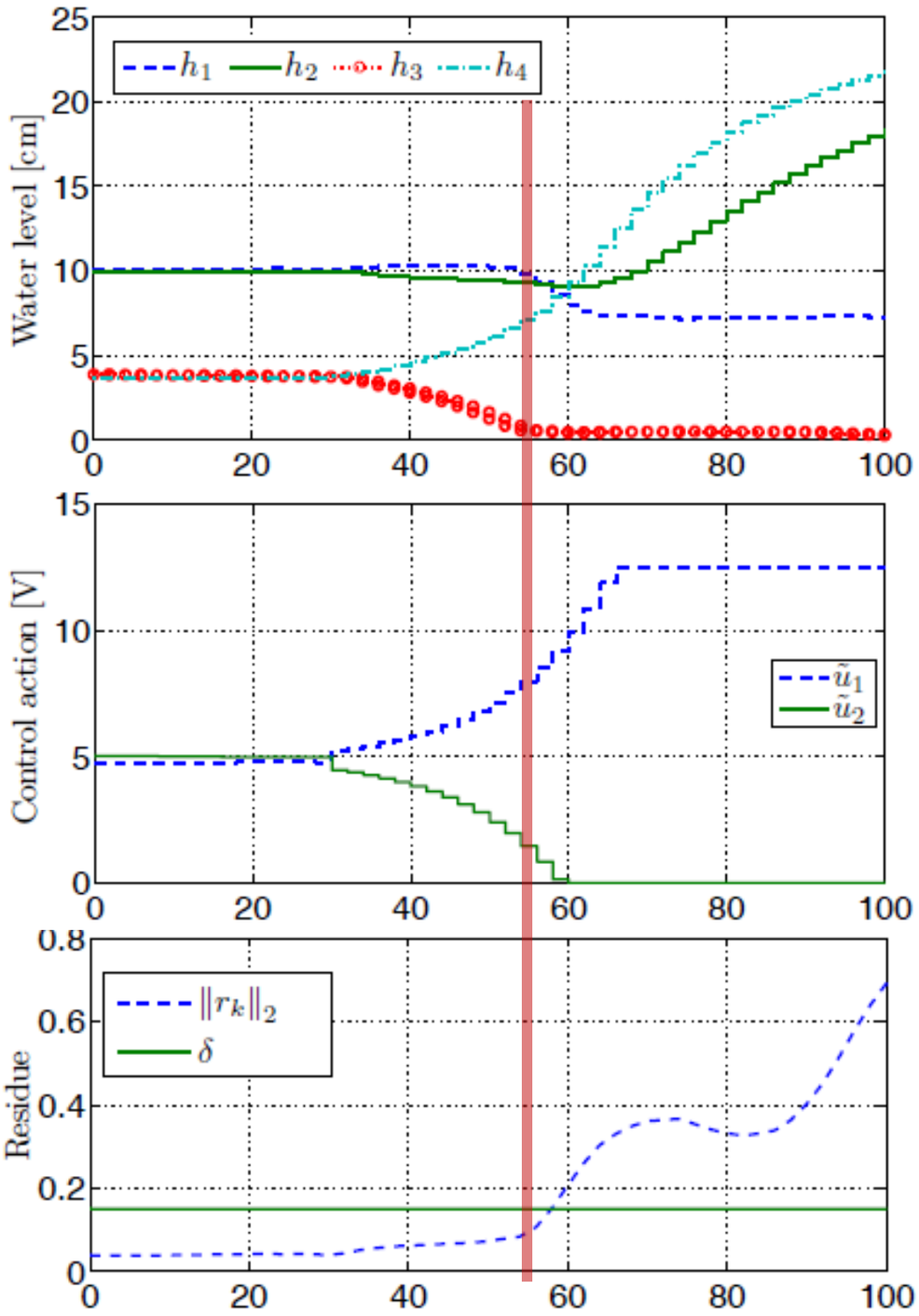

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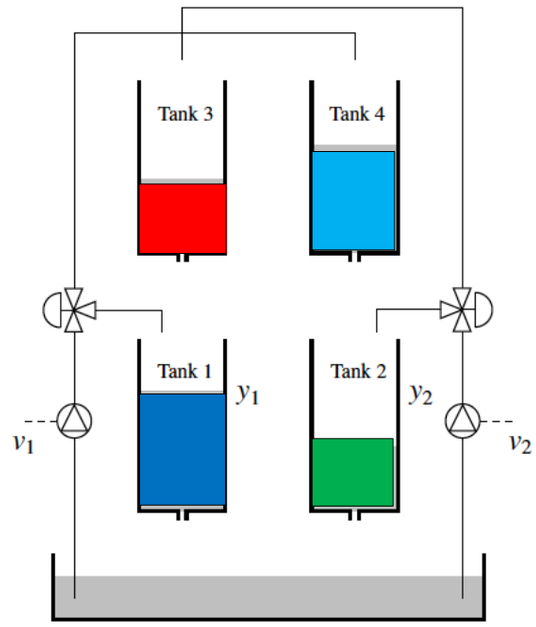
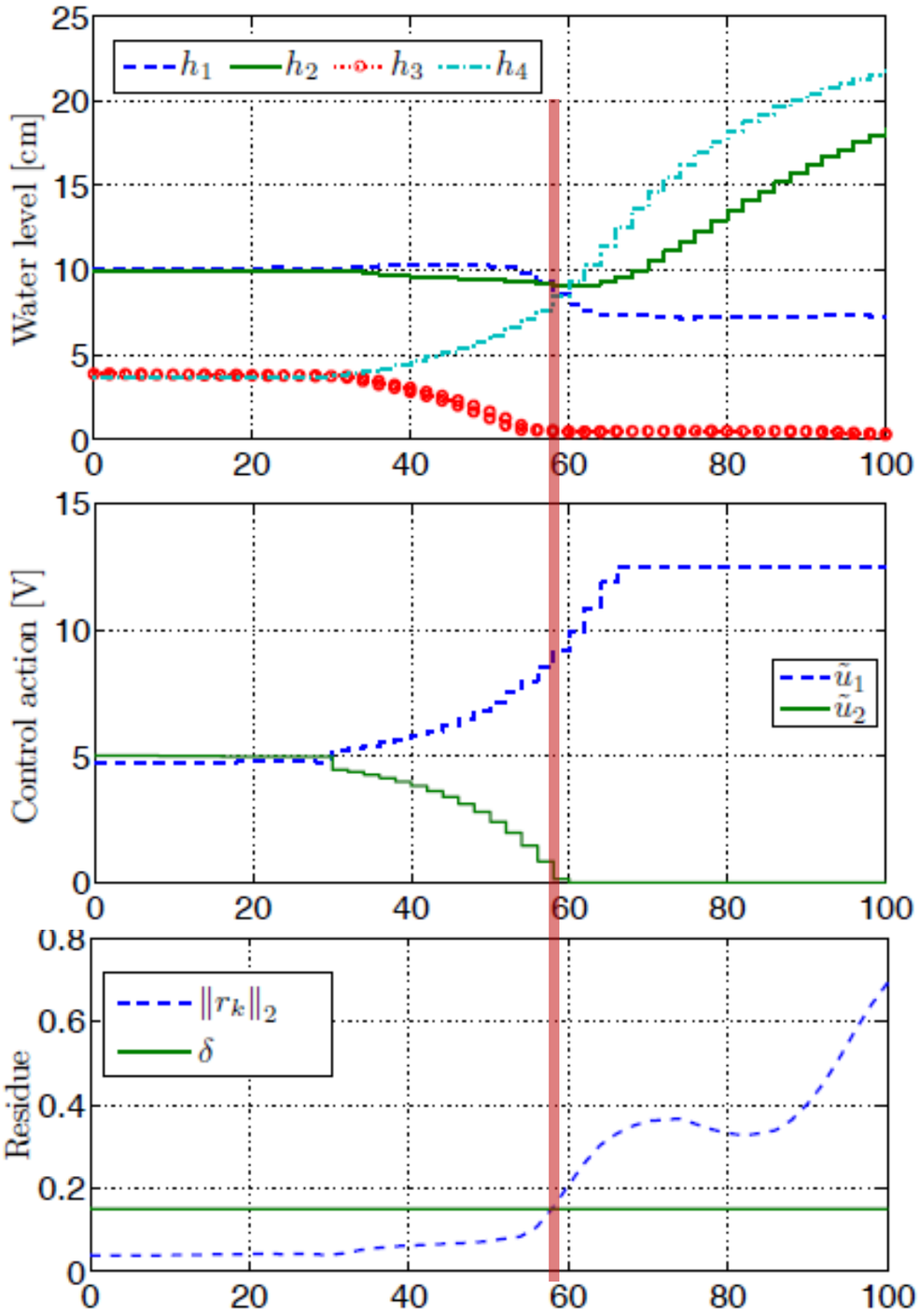




Attack Goal: Empty tank 3

- Zero dynamics attack on both actuators - unstable zero
- Tank 3 becomes empty
- The attack is **detected** (Why? Can we exploit this for detection?)

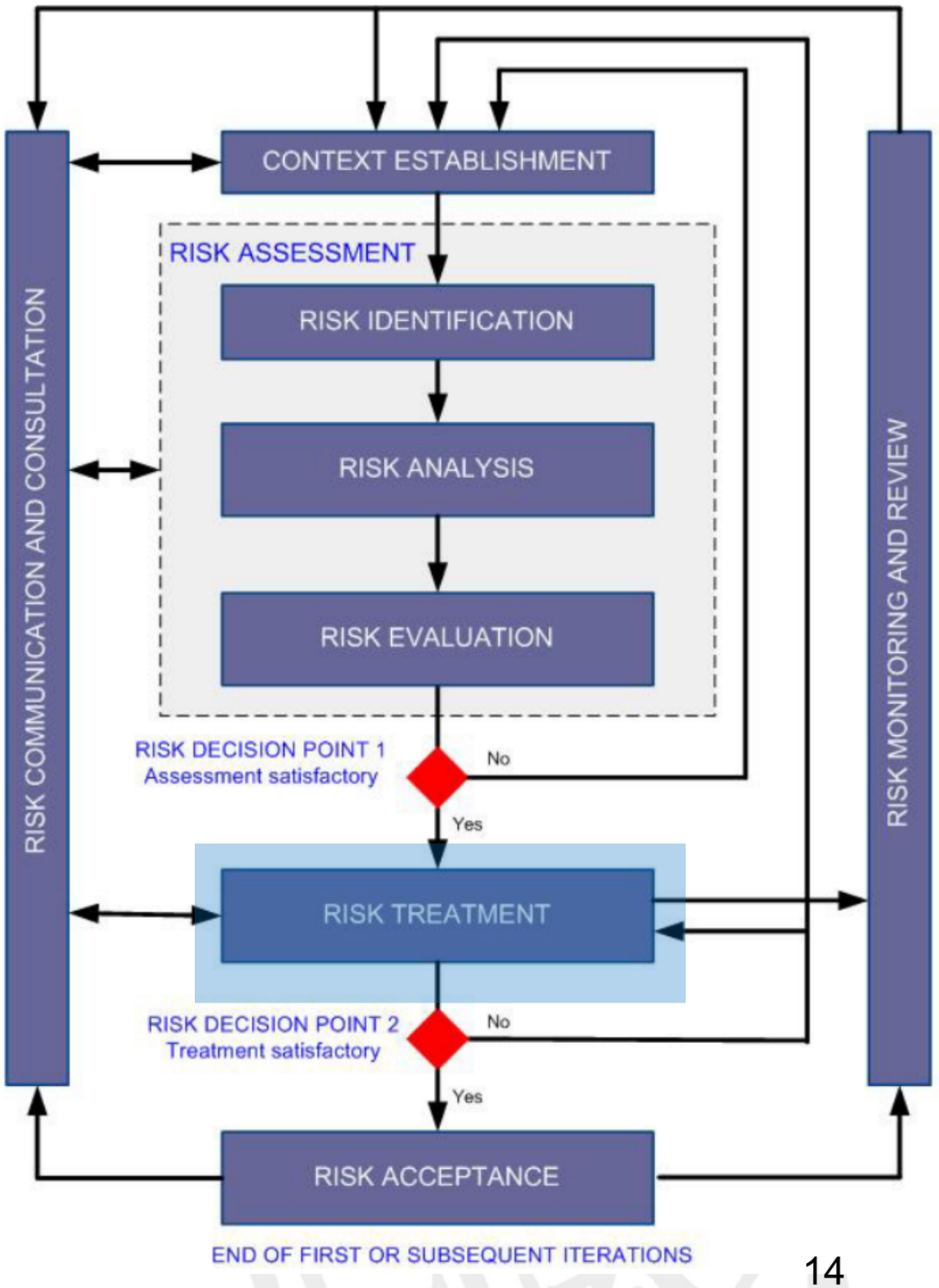
Risk Management Cycle

Risk = (Scenario, Likelihood, Impact)

[Kaplan & Garrick, 1981]

Main steps in risk management

- Scenario characterization
  - Models, Scenarios, Objectives
- Risk Analysis
  - Likelihood Assessment
  - Impact Assessment
- **Risk Mitigation**
  - **Prevention, Detection, Treatment**



[ISO 31000]

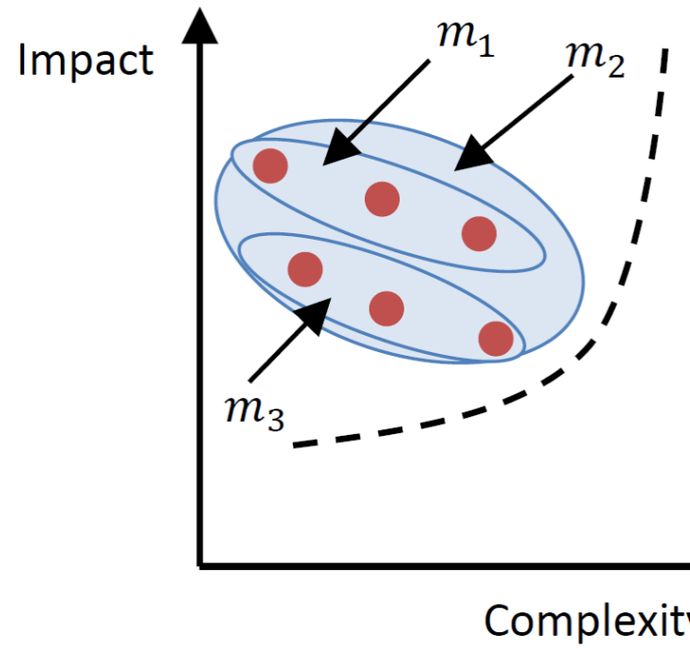
Overview - Risk Mitigation

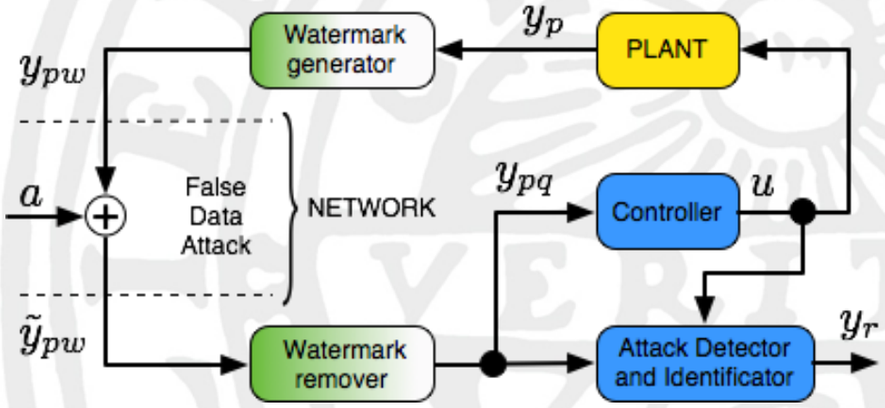
- **Prevention** (decrease likelihood by reducing vulnerability)
  - Watermarking and Moving Target Defense
  - Coding and Encryption Strategies
  - Rational Security Allocation
  - Privacy-preservation by Noise Injection

- **Detection** (continuous anomaly monitoring)
  - Tuning of Detector Thresholds
  - Secure State Estimation
  - Watermarking and Moving Target Defense
  - Distributed Algorithms
  - Methods Related to Robust Statistics

- **Treatment** (compensate for or neutralize detected attack)
  - Secure State Estimation
  - Resilient Control Countering DoS Attacks
  - Distributed Algorithms
  - Methods Related to Robust Statistics

[Chong *et al*., ECC, 2019]





[Ferrari and Teixeira, TAC 2021]

Control Theoretic View

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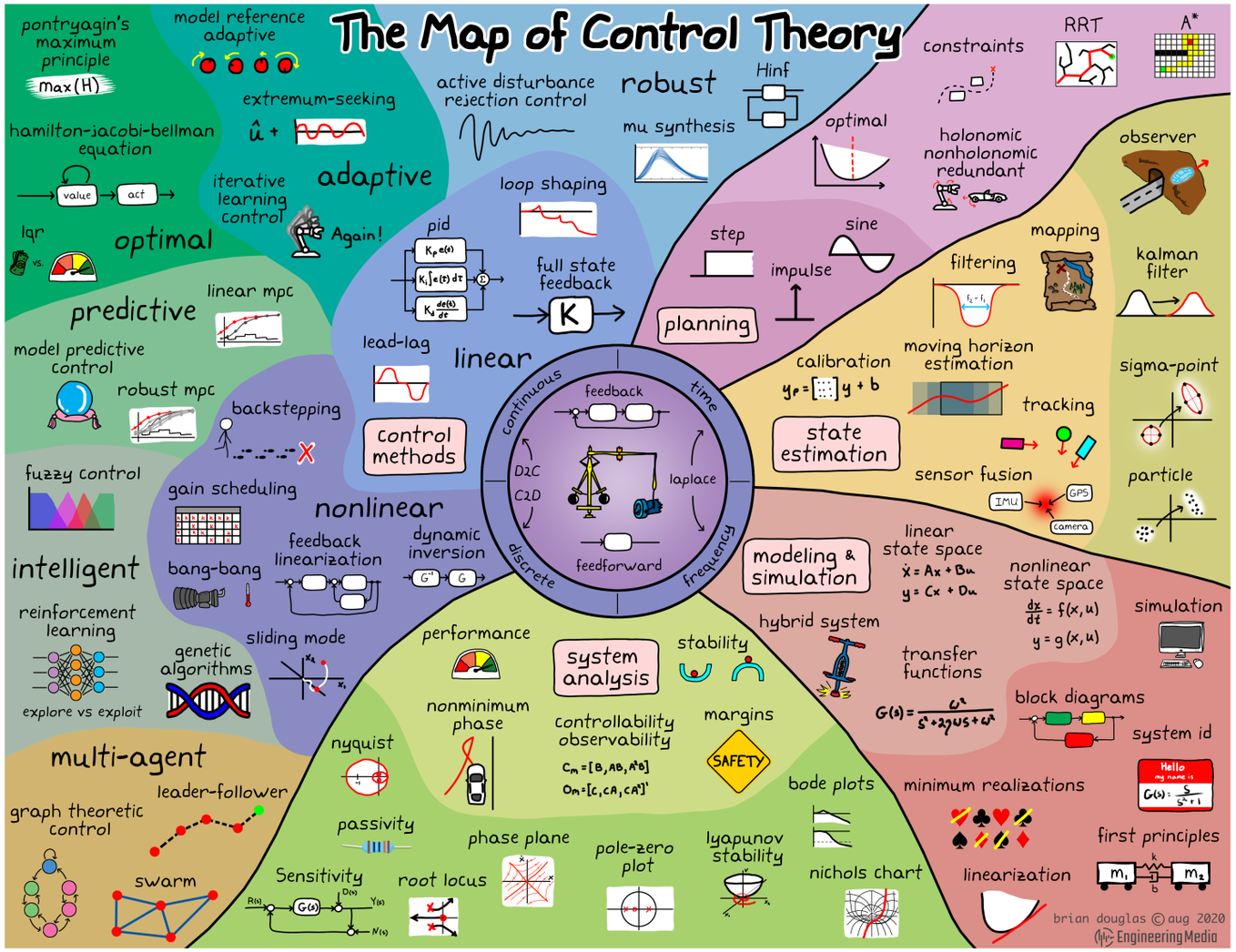

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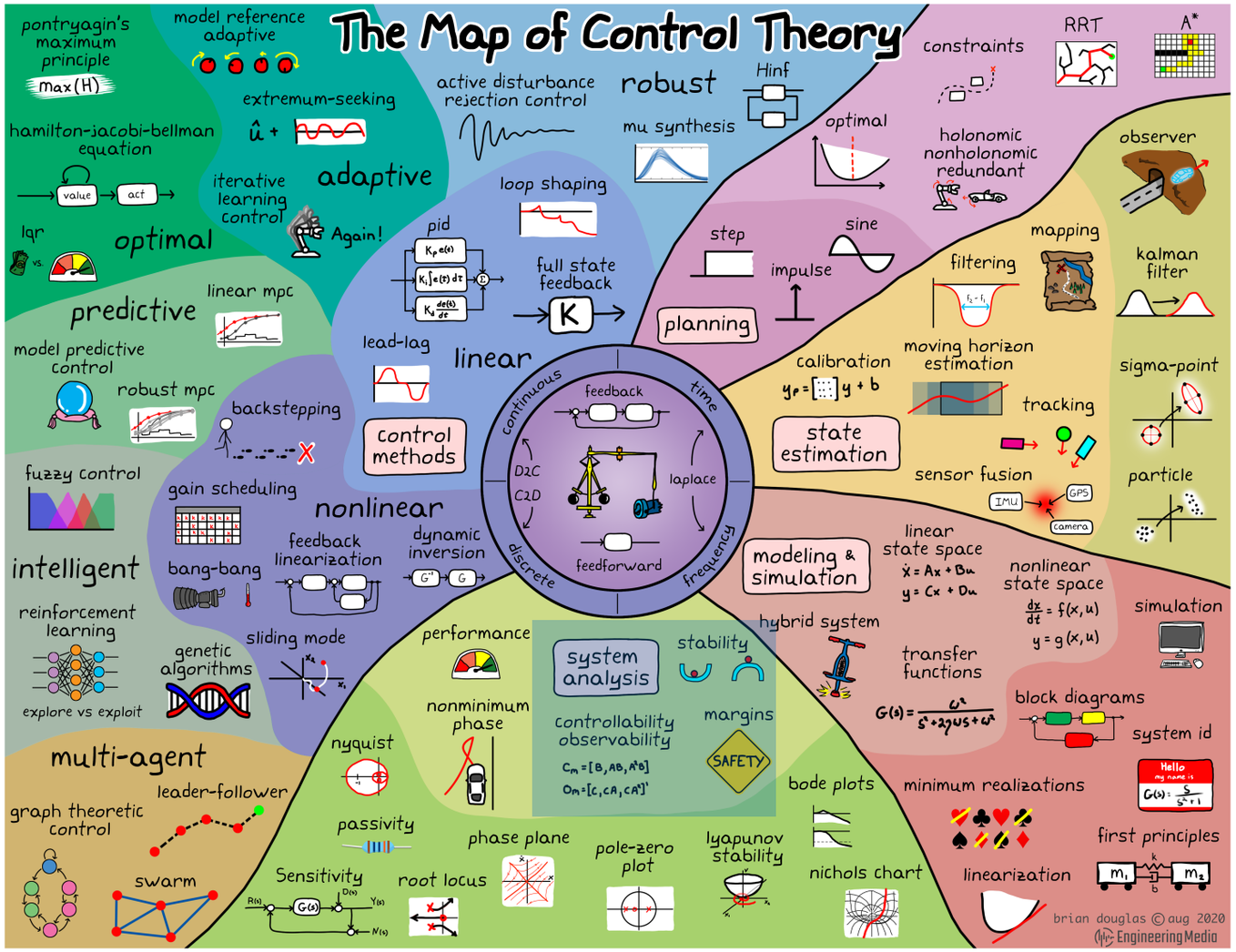

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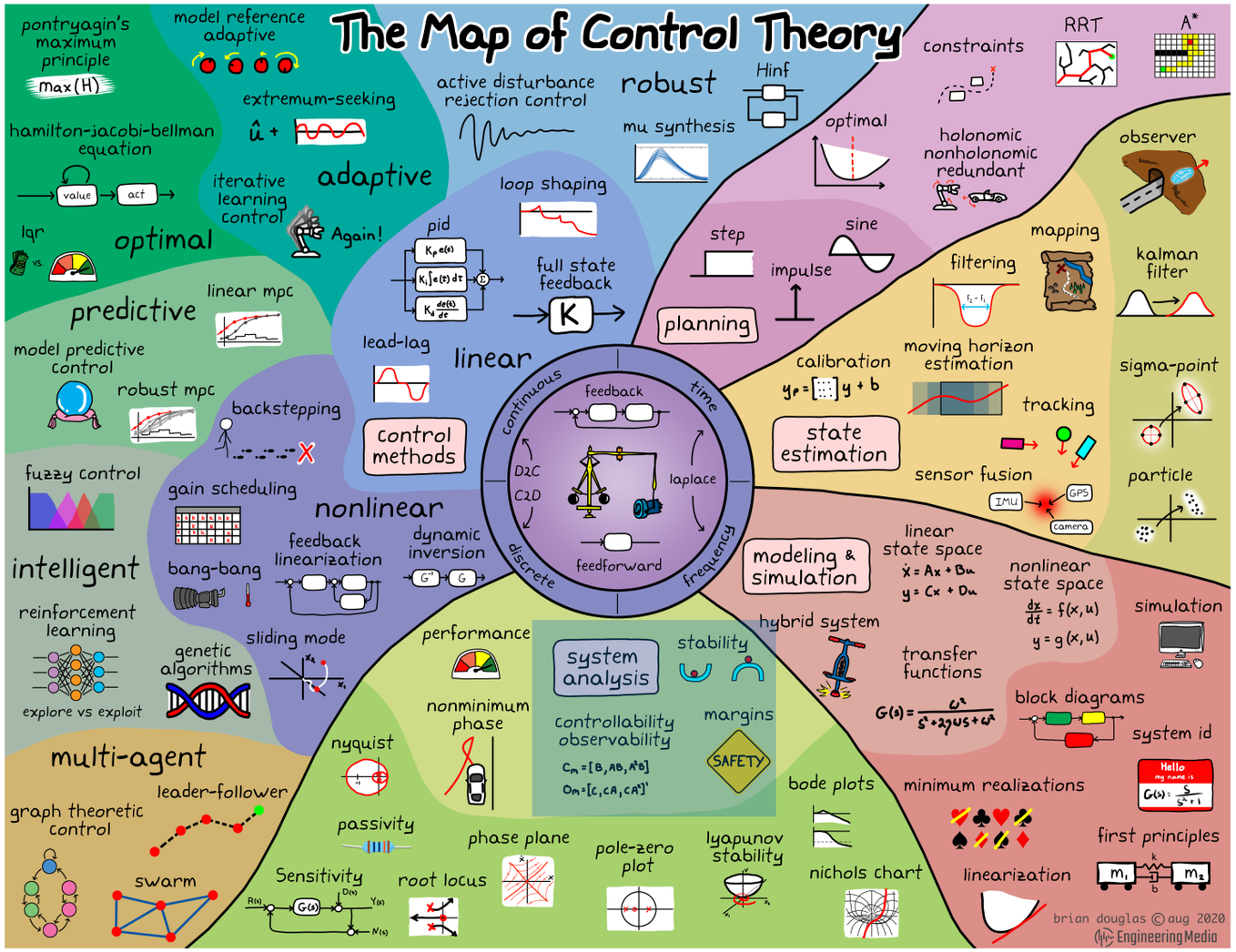

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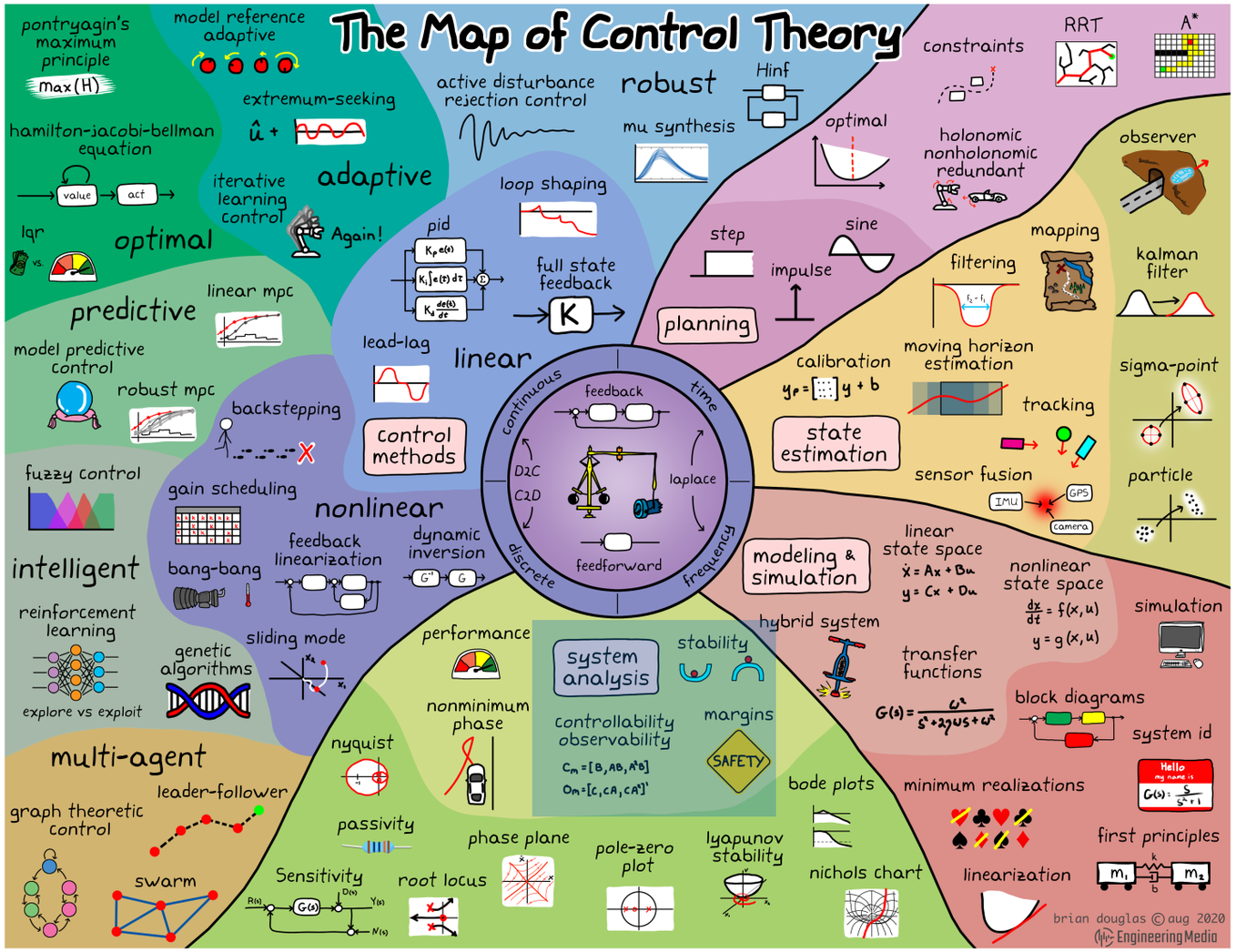

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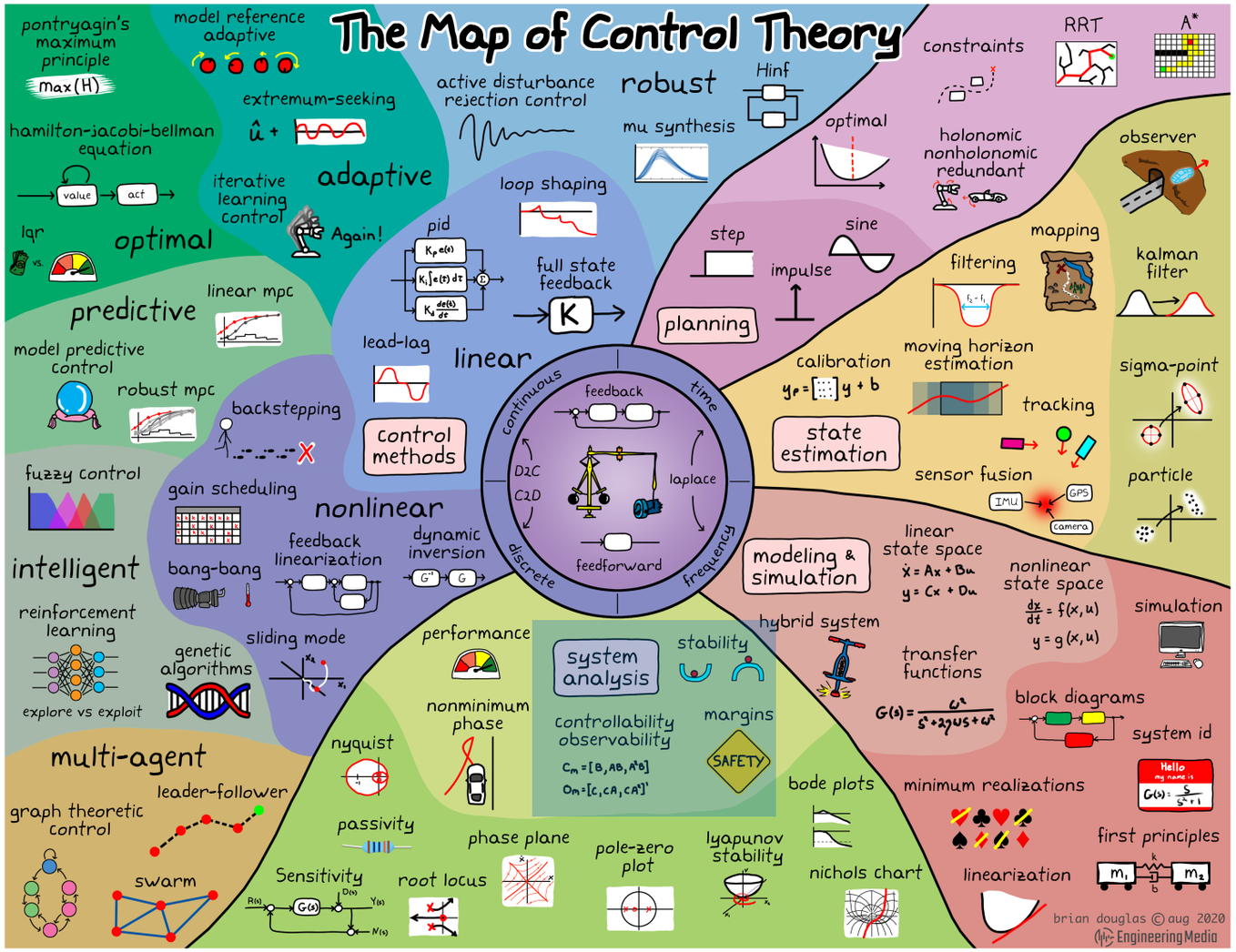

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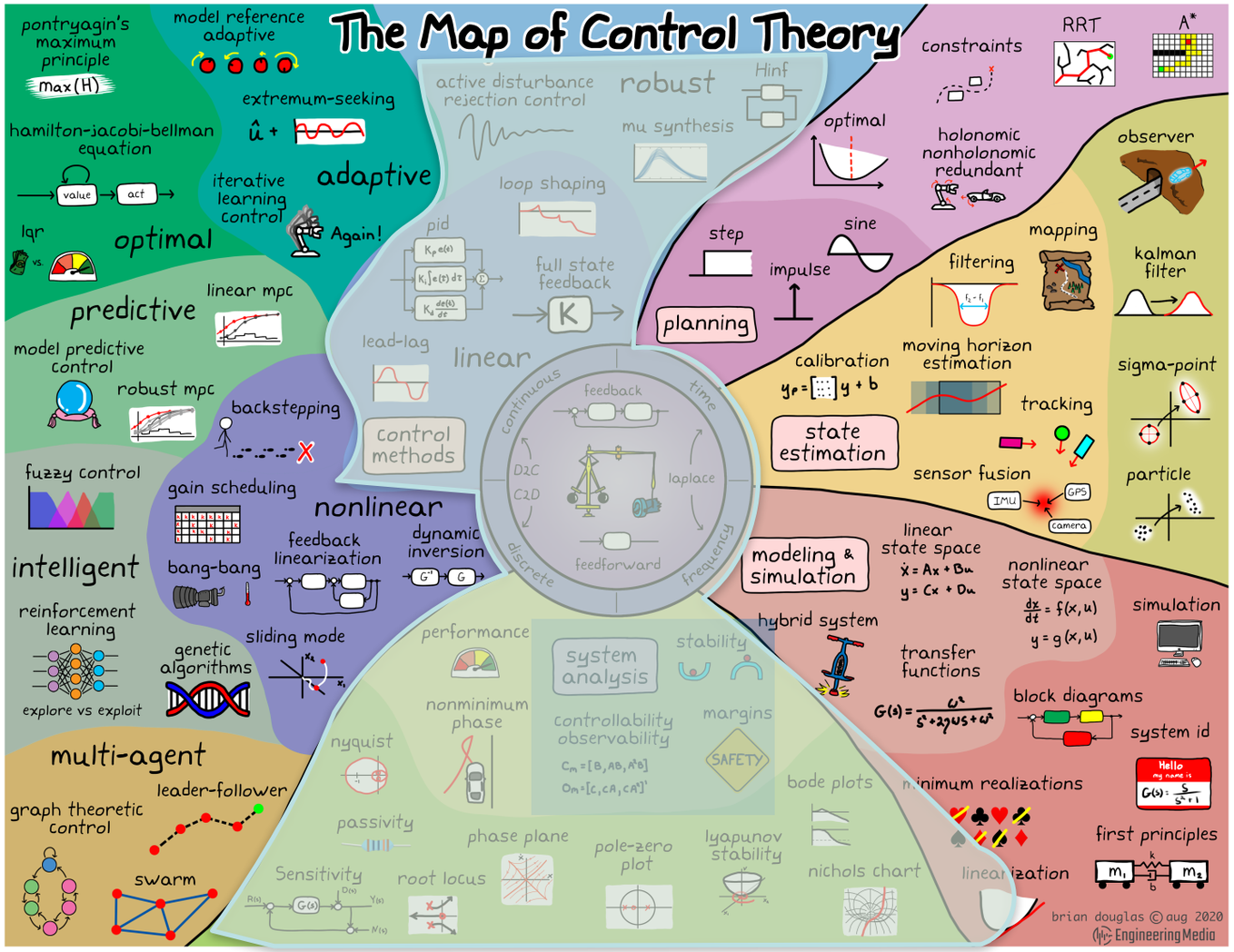

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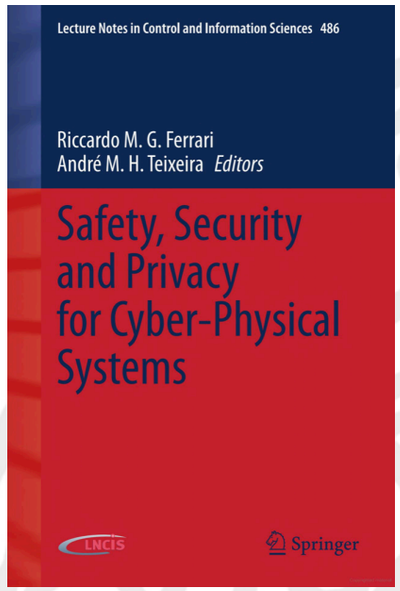
Outline

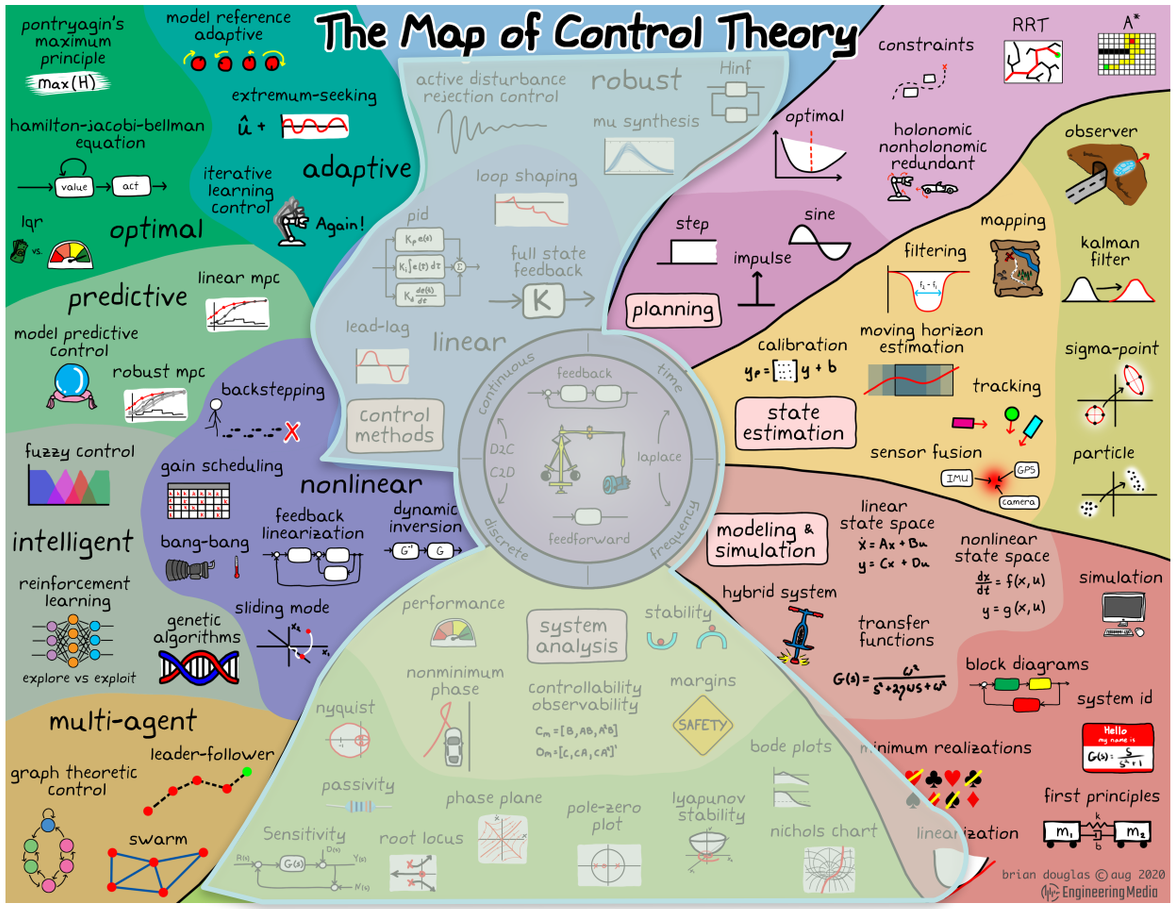
- Cybersecurity in Control Systems
  - Security Game
  - Risk Management
    - Mapping to Control System Design
    - Example: undetectable attacks

- Security Metrics for Control Systems
  - Classical metrics in control engineering
  - Novel security metric for analysis and design
    - Analysis and design problems
    - Structural limitations - invariant zeros

- Additional topics
  - Variations of the adversary models and their respective metrics
  - Incorporating uncertainty for robust open-loop attacks



Control System Design



Main steps in control system design:

- Scenario characterization
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  - Sensitivity metrics (w.r.t. input) [not binary in general]

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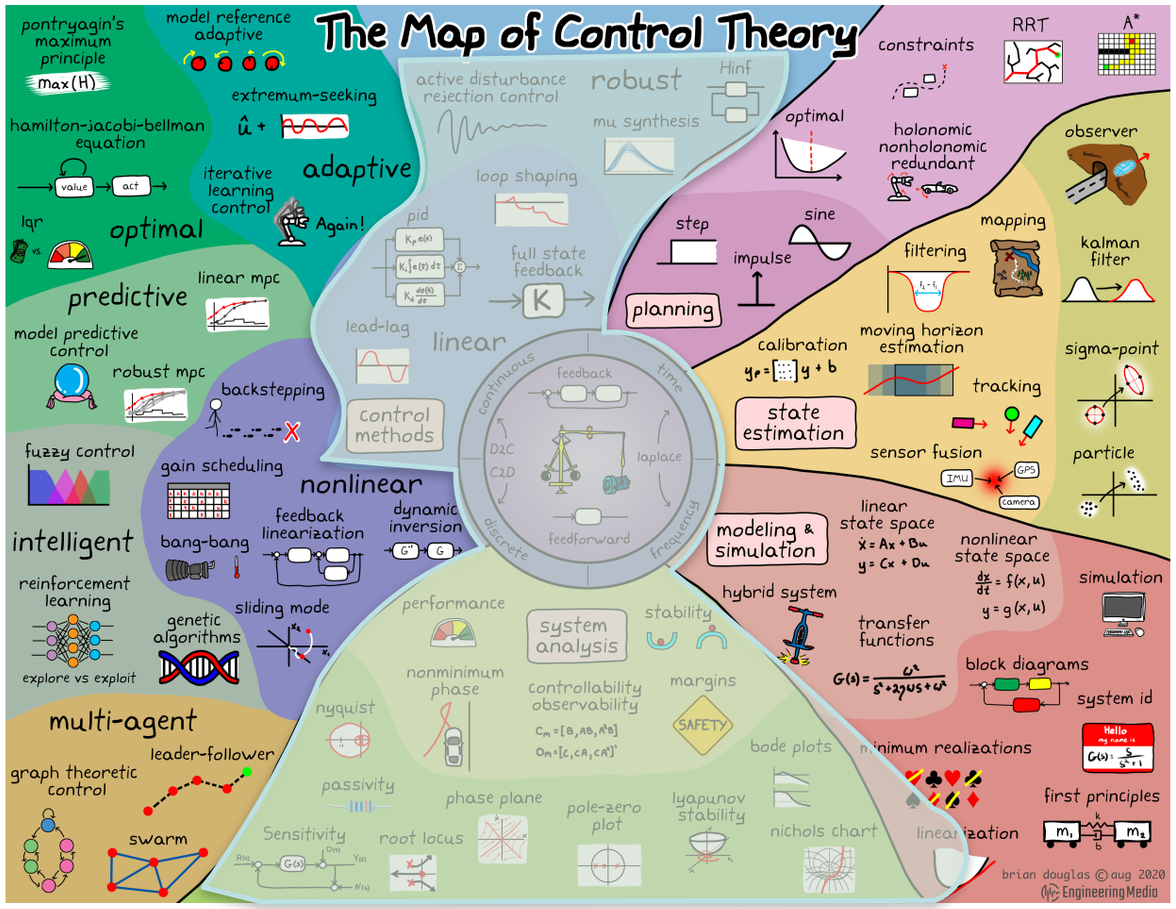

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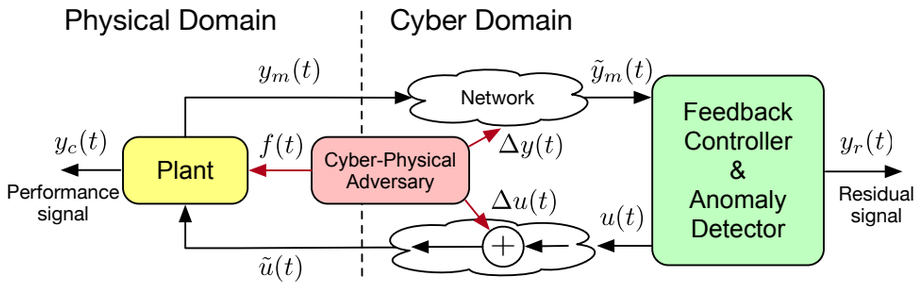

Adversary Model

$$y_p(t)$$

$$\dot{x}(t) = Ax(t) + Ba(t)$$
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$a(t)$

$$y_r(t)$$

Closed-loop system

Model knowledge

$$\{\Sigma_p,\ \Sigma_r\}$$

Disruption Resources

$a(t)$

Attack Policy

$$\Sigma_p = (A, B, C_p, D_p),\ G_p(s) = C_p(sI - A)^{-1}B + D_p$$
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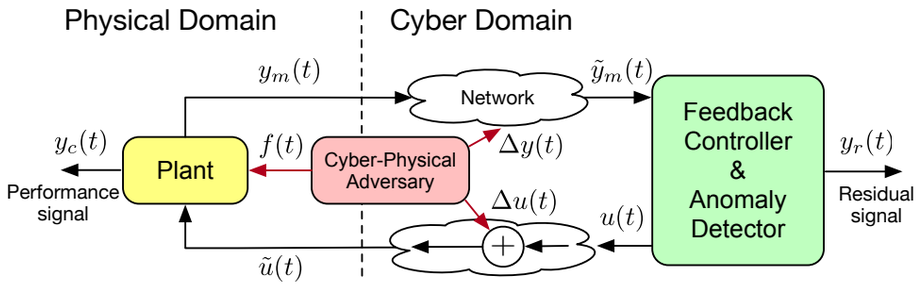

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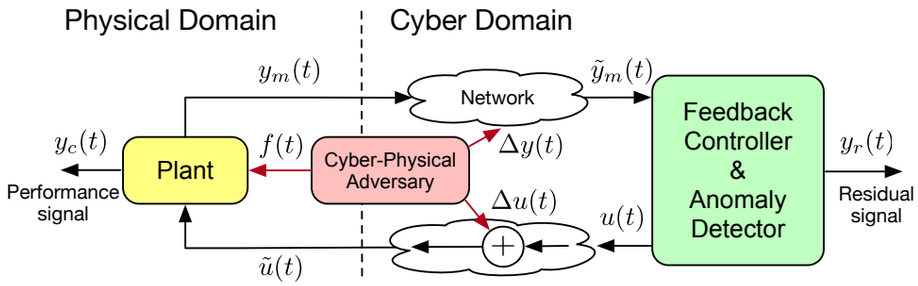

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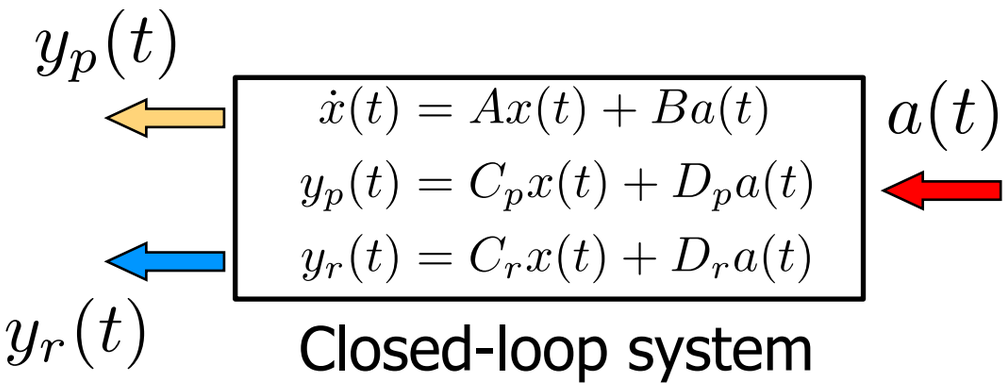

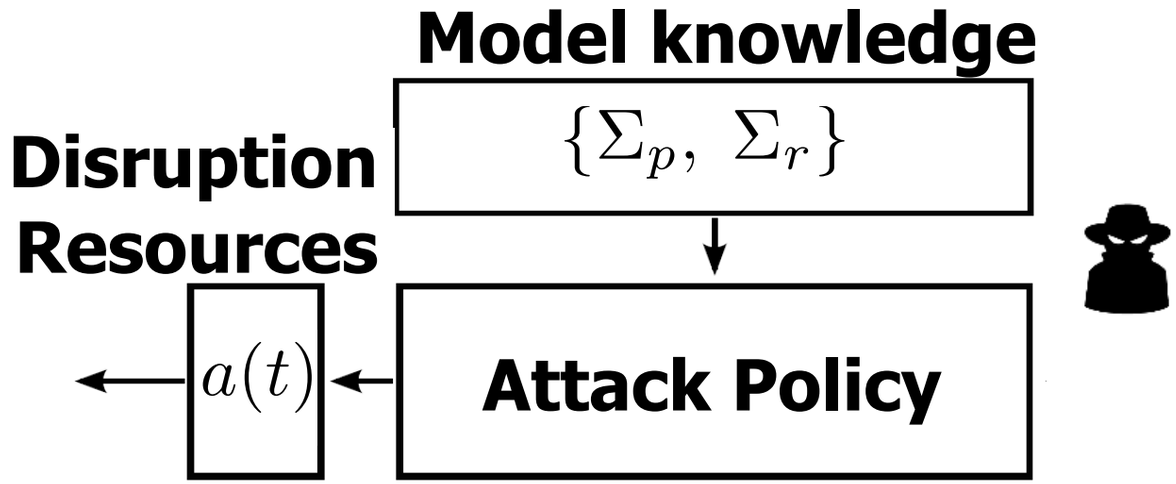

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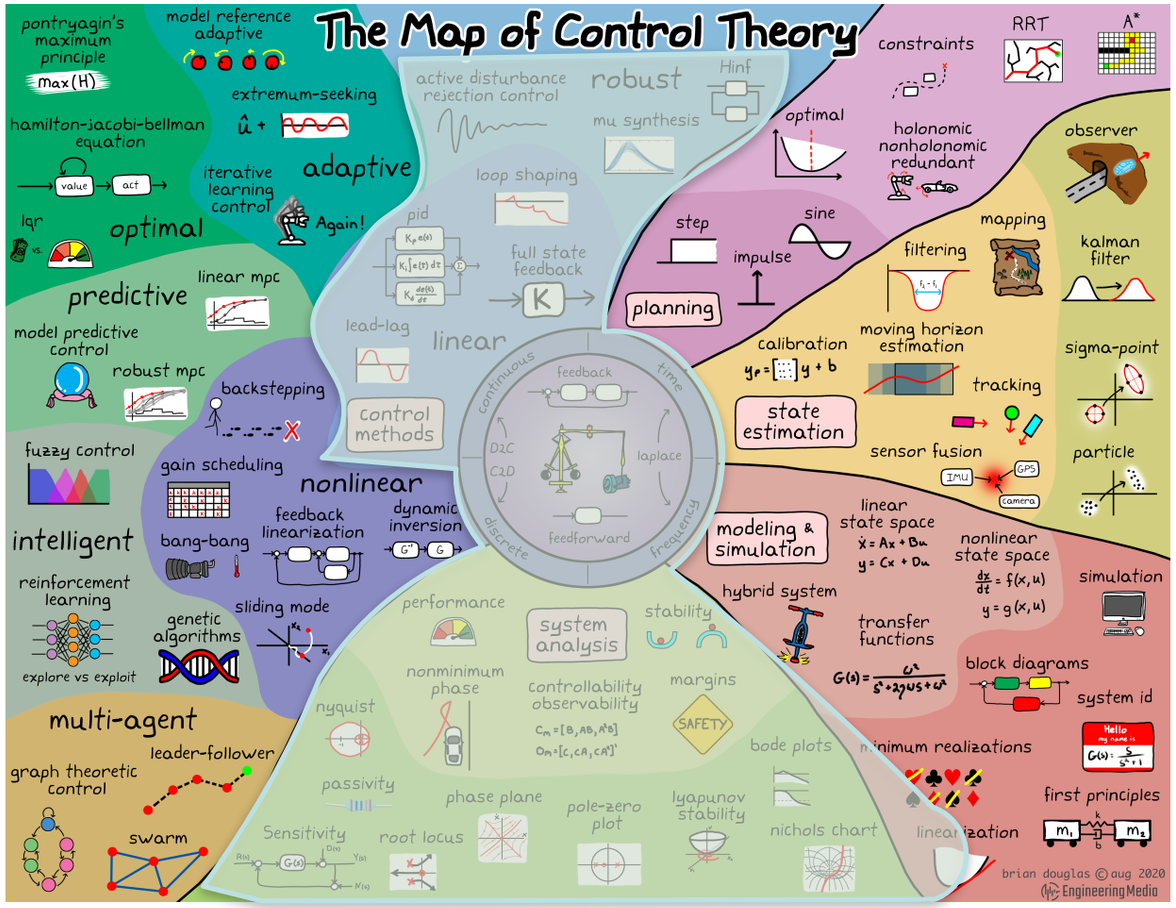
How to analyse and design the system with such an attack?

Control System Design



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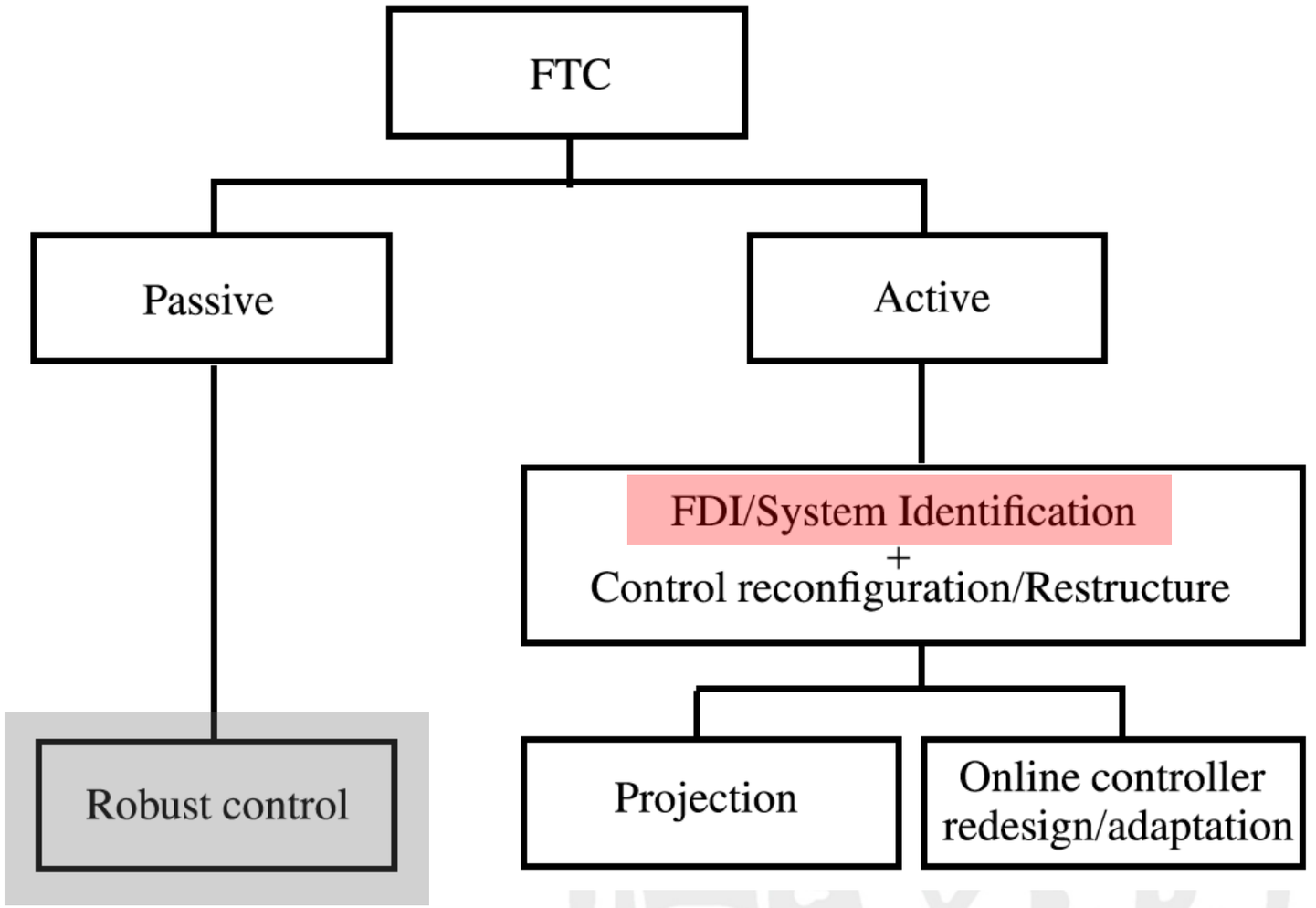
Fault-Tolerant Control

- While faults are not *detected*:
  - *Robust controller*
- If a fault is *detected*:
  - adapt or redesign
- Key fields of interest for "weakly detectable" attacks:
  - Fault Detection
  - Robust Control



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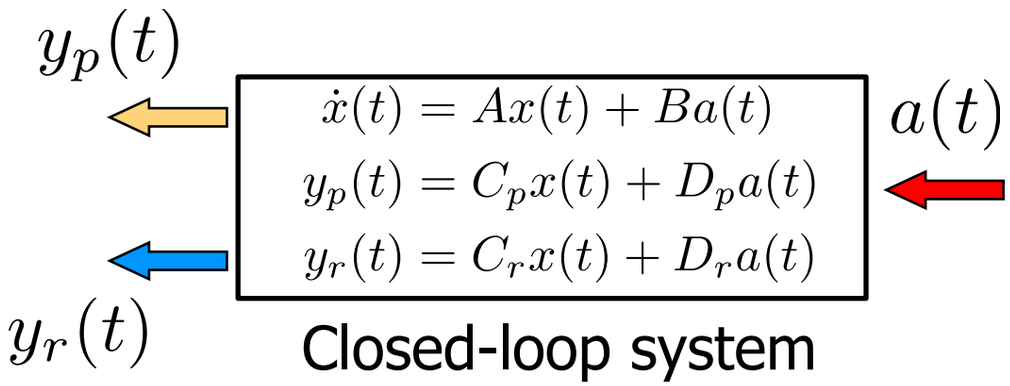
- Both are based on *sensitivity metrics*:
  - **Robustness**: largest **impact on performance** of unit-energy faults
  - **Detectability**: smallest **detectability** of unit-energy faults

Classical Sensitivity Metrics

$\mathcal{L}_{2e}$ = "signals with finite energy over finite time intervals"

$$\|y\|_{\mathcal{L}_2}^2 \triangleq \int_{-\infty}^{+\infty} \|y(t)\|_2^2 \, dt$$



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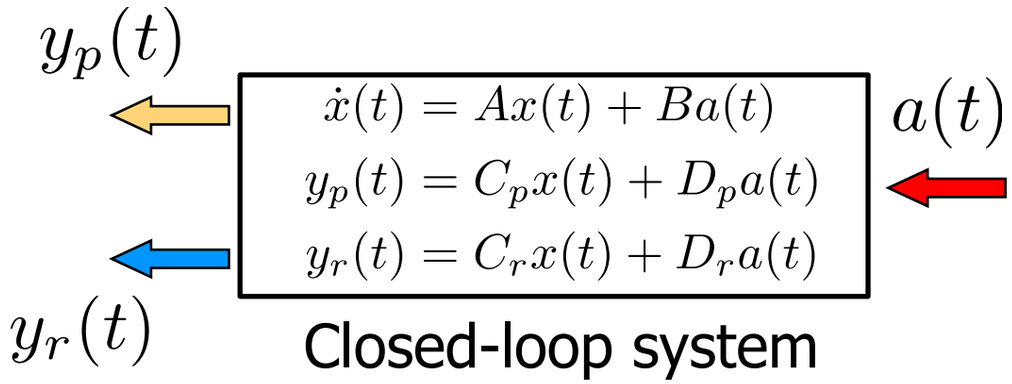

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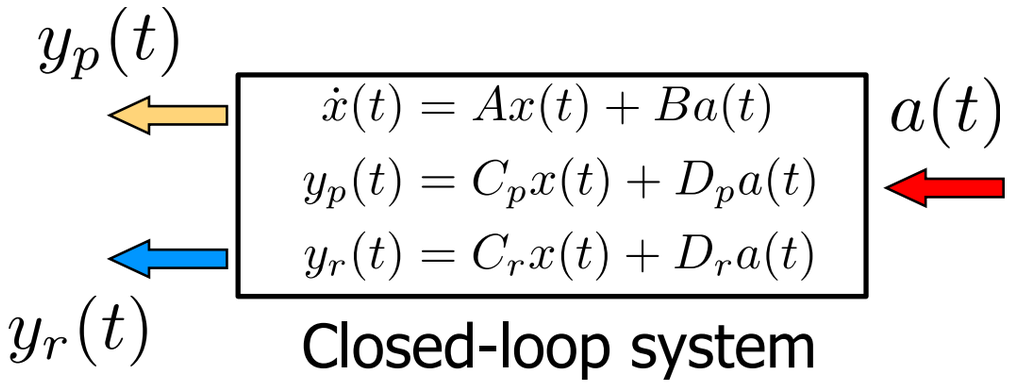

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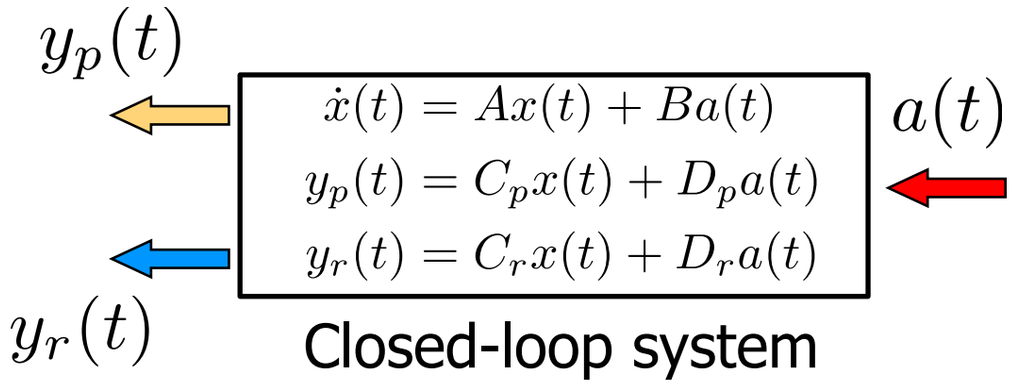

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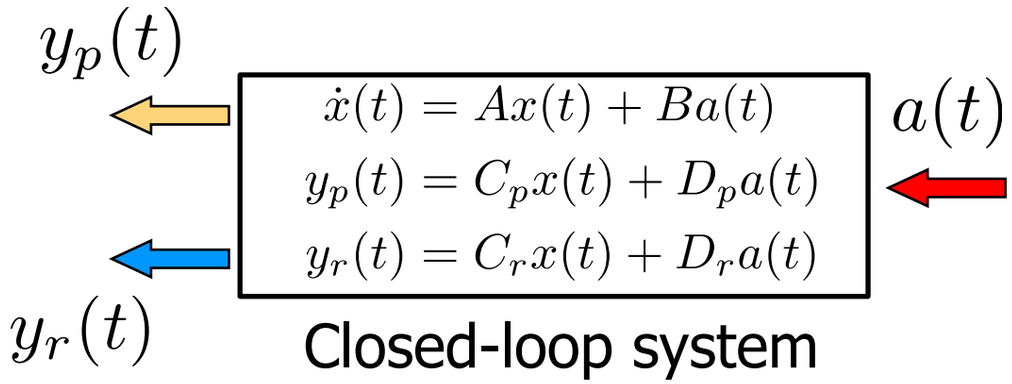

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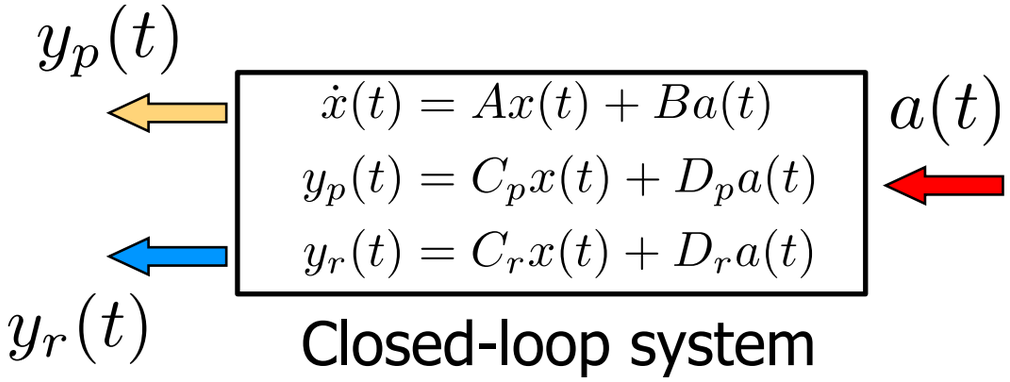

- Robustness (H_∞)

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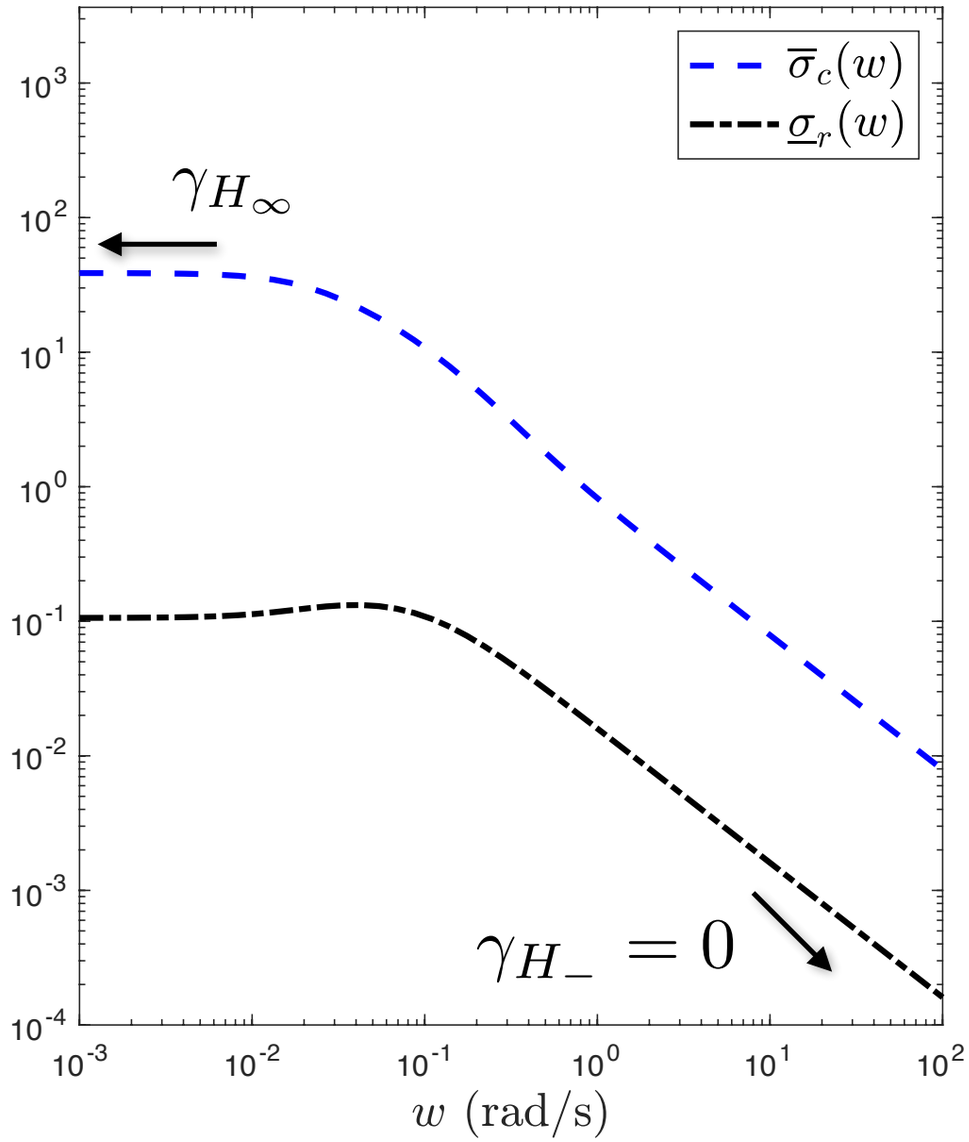

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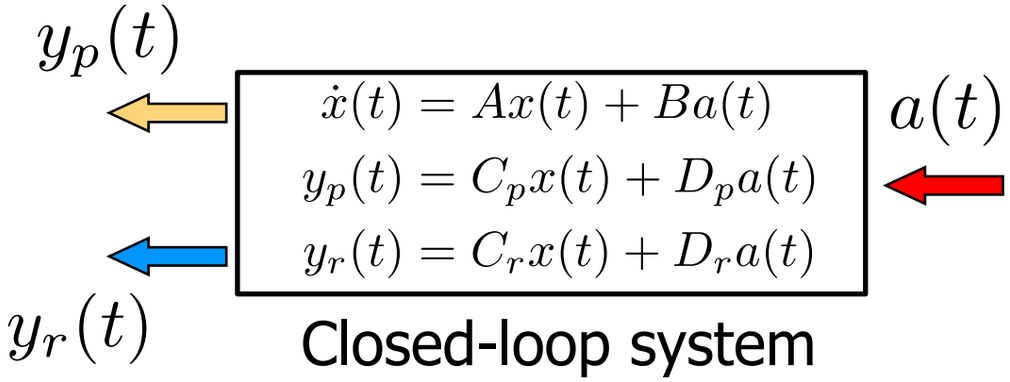

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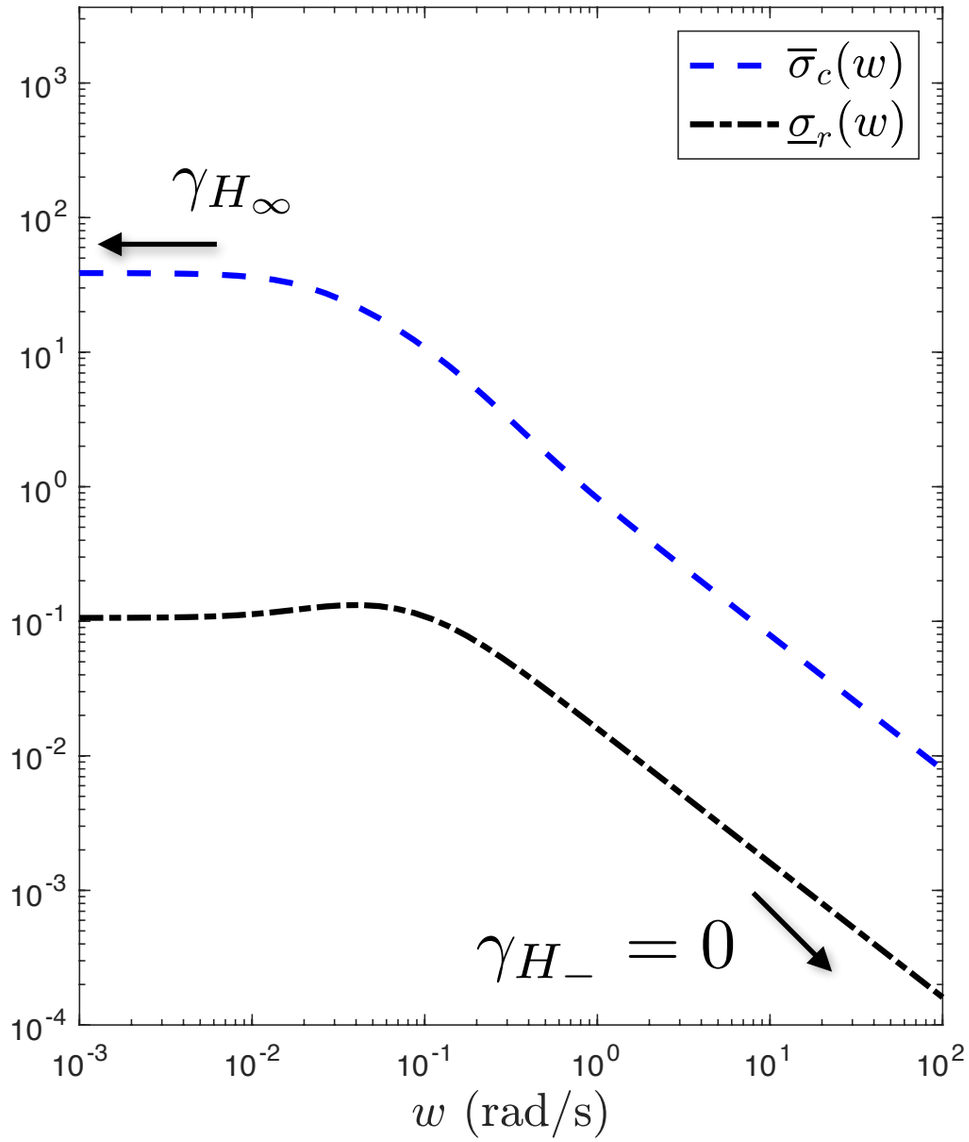

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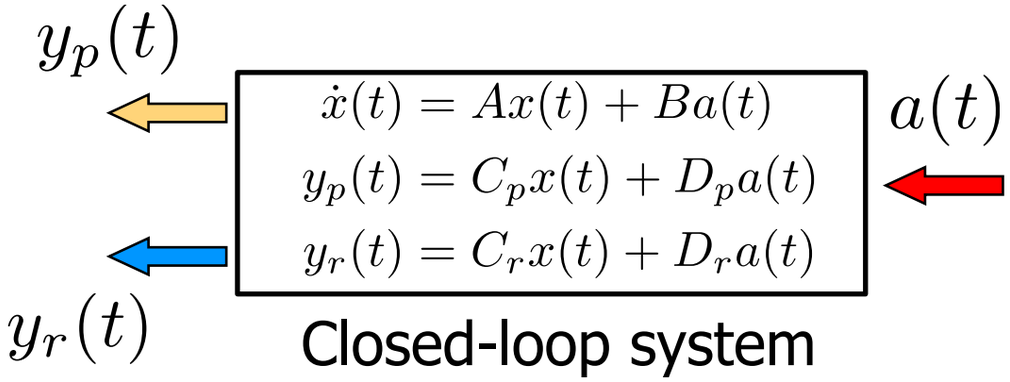

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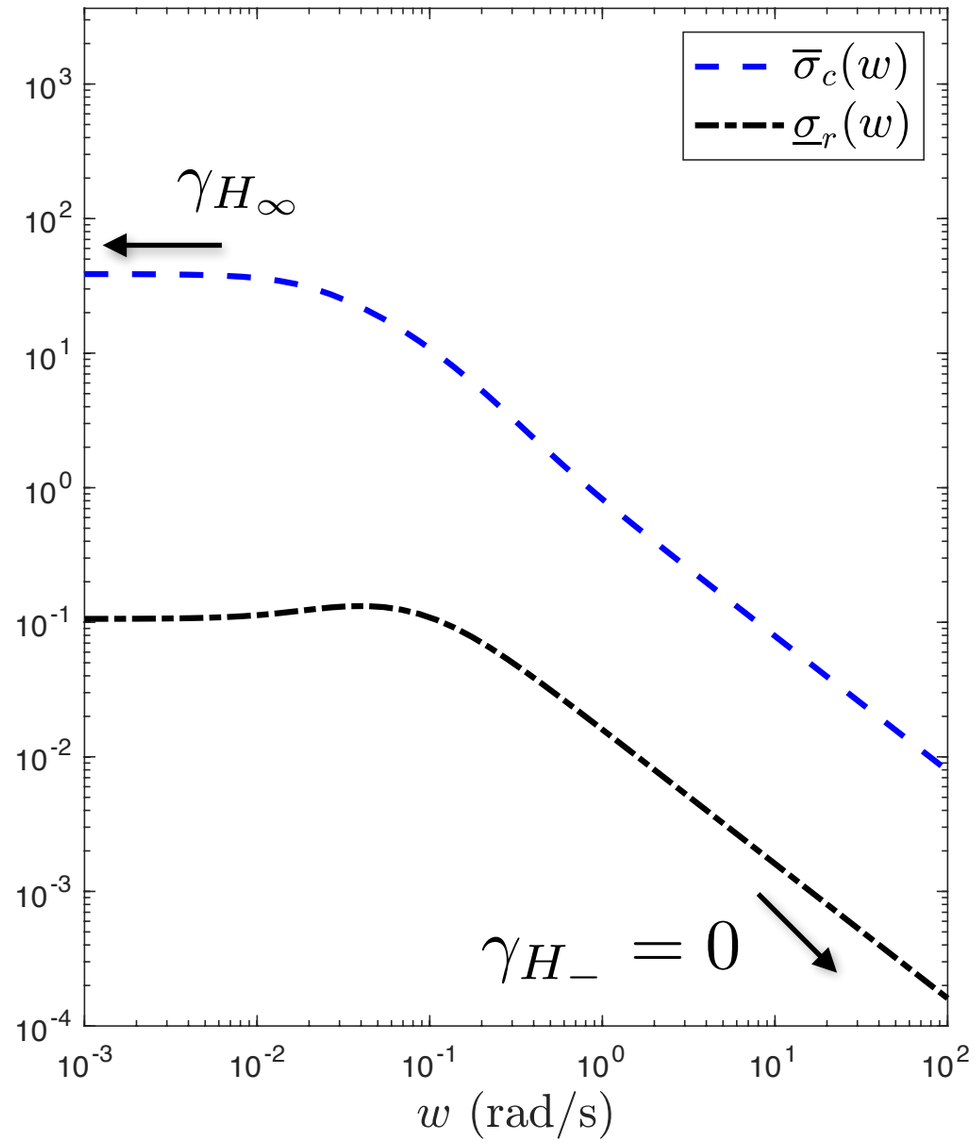

- **Limitation:** worst-case frequency is not the same
  - Each metric looks at **different** worst-case inputs!

Security metrics for control systems (a first heuristic)

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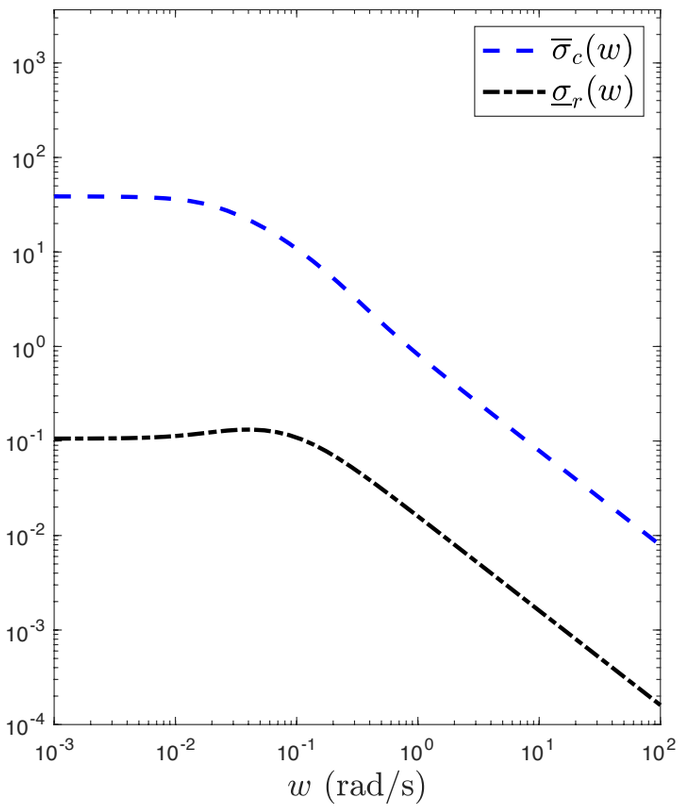

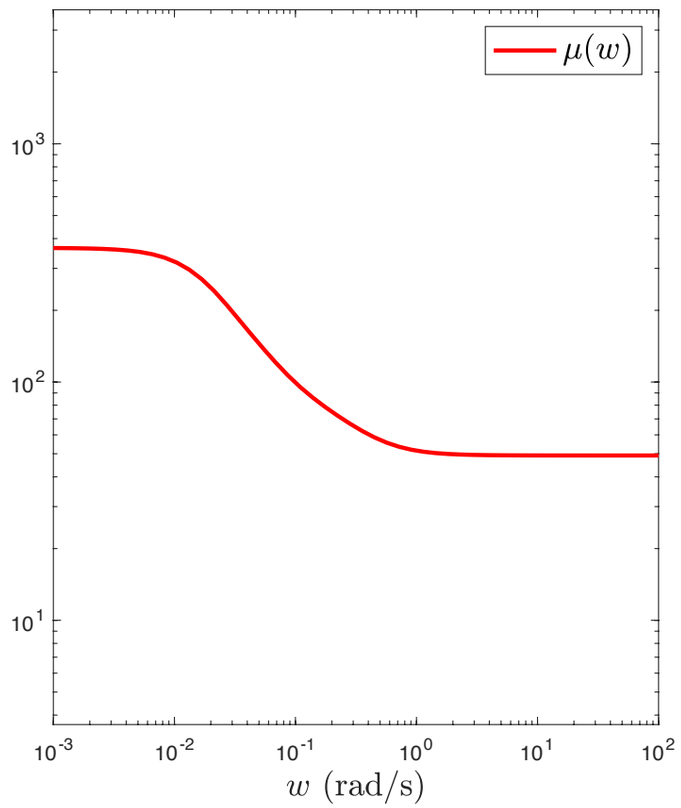

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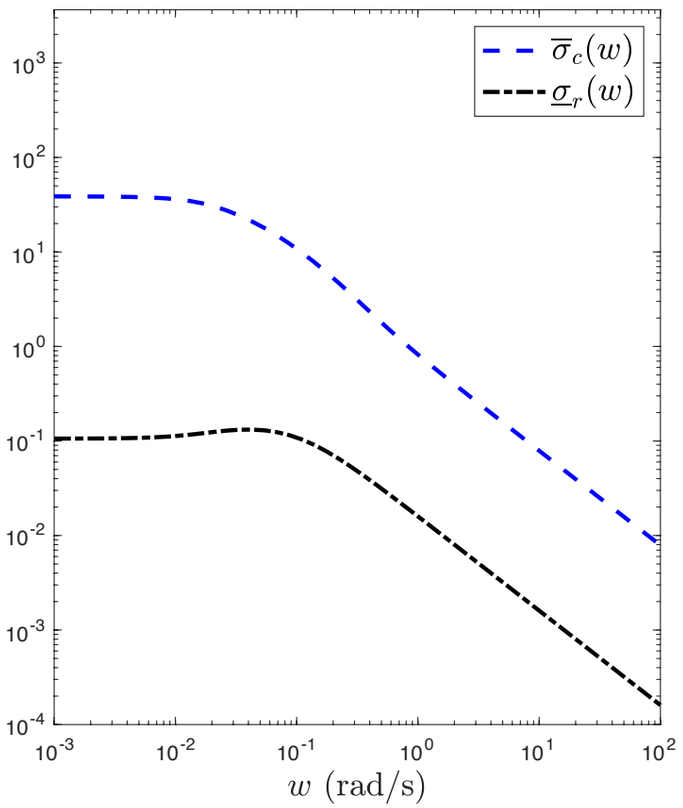

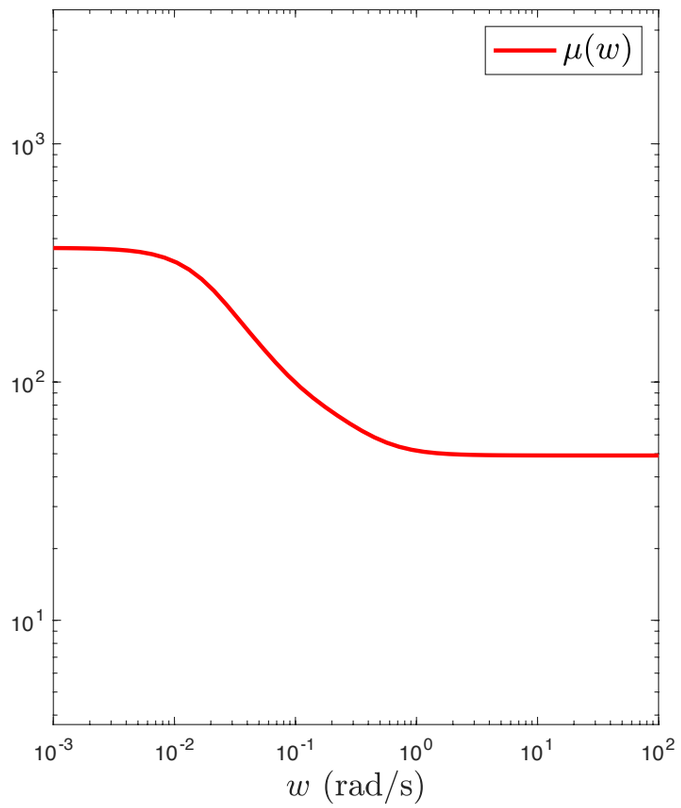

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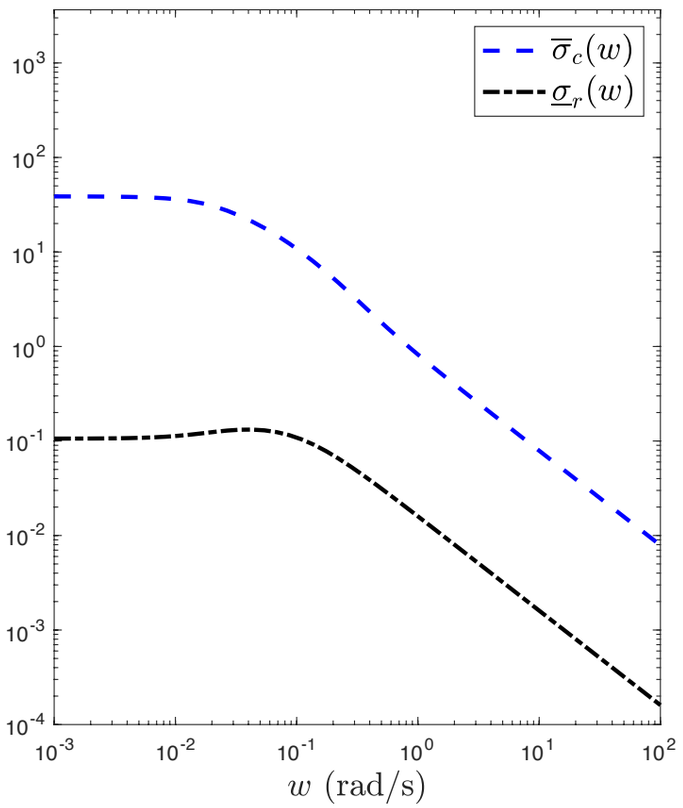
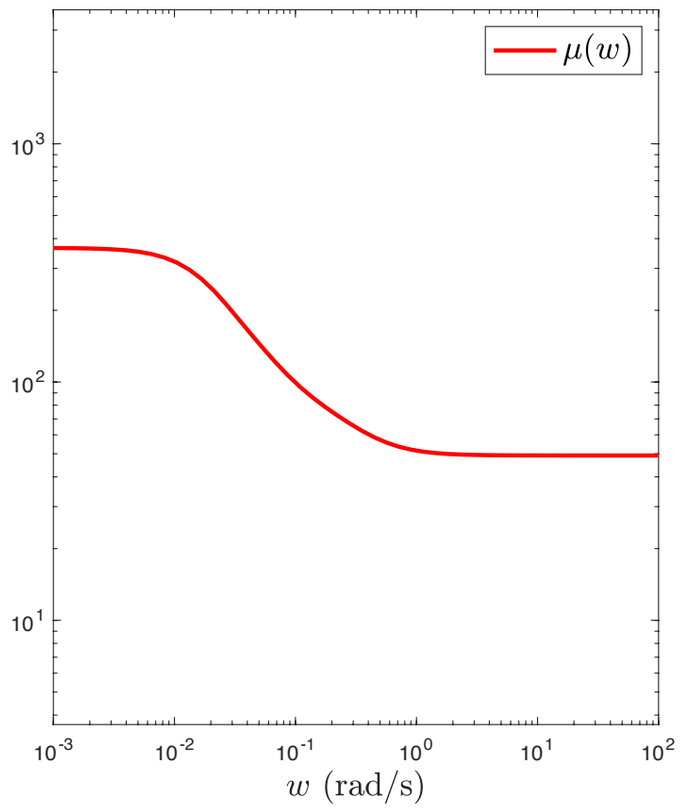
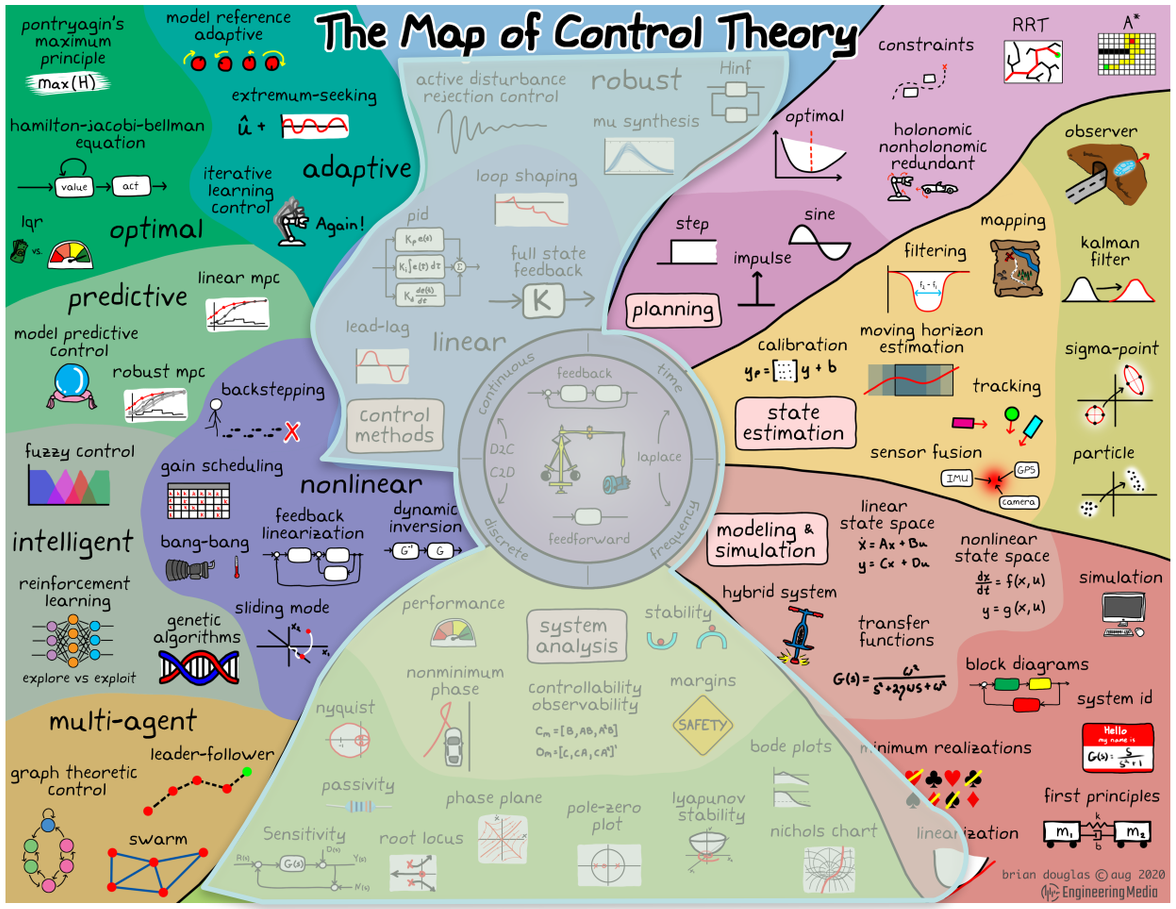




- But this is an heuristic…
- And how do we formalise such a metric?
  - … coming next!

Control System Design



Main steps in control system design:

- Scenario characterization
  - Models, Scenarios, Cost functions
- System Analysis
  - Sensitivity metrics (w.r.t. input)
  - **<u>Need to capture security aspects:</u>** maximise impact while undetected
- System Design (Control Methods)
  - Find control and monitoring algorithms that <u>minimise the sensitivity metric</u>
  - Re-design the structure of the system to <u>minimise the sensitivity metric</u>

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- Note: Input is not directly constrained
  (may be exponentially increasing)

$\mathcal{L}_{2e}$ = "signals with finite energy over finite time intervals"

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- Finite $\gamma^\star$ implies a bound on the performance degradation by undetectable attacks

$$\|y_r\|_{\mathcal{L}_2}^2 \leq \theta \rightarrow \gamma^{\star^2} \theta \geq \|y_p\|_{\mathcal{L}_2}^2$$

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Control System Analysis: Linear Matrix Inequalities

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- Key technique: Dissipative Systems Theory

[Teixeira, CDC 15]

Dissipative Systems Theory

Consider the LTI system Σ with input a and outputs y_p and y_r. The following statements are equivalent:

1. the system Σ is dissipative w.r.t. $s(a,x) = \beta\|y_r(t)\|_2^2 - \|y_p(t)\|_2^2$;
2. for all trajectories of the system such that $T > 0$ and $x(0) = 0$, we have $\int_0^T \|y_p(t)\|_2^2 \leq \beta \int_0^T \|y_r(t)\|_2^2$;
3. there exists a positive semi-definite matrix $P \succeq 0$ such that the following linear matrix inequality (LMI) holds:

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- *Note of caution:* in general, there is no simple equivalent frequency domain inequality

H.L. Trentelman. When does the algebraic Riccati equation have a negative semi-definite solution?. In: Blondel, V., Sontag, E.D., Vidyasagar, M., Willems, J.C. (eds) Open Problems in Mathematical Systems and Control Theory. Communications and Control Engineering. Springer (1999)

On the Frequency Domain Inequality (extra)

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  - Not in general, but there is a necessary condition for the LMI to be feasible

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  - see H_∞ norm (and the “Periodic Attacks” in later slides).
- Necessary condition relates to generalised singular values: $\mu(s) \triangleq \min_{\beta} \quad \sqrt{\beta}$

$$\text{s.t.} \quad \beta G_r(\bar{s})^\top G_r(s) - G_p(\bar{s})^\top G_p(s) \succeq 0$$

On the Frequency Domain Inequality (extra)

$$\gamma^{\star^2} = \min_{\beta \geq 0,\, P \succeq 0} \quad \beta$$

$$\text{s.t.} \quad \begin{bmatrix} A^\top P + PA & PB \\ B^\top P & 0 \end{bmatrix} - \beta \begin{bmatrix} C_r^\top C_r & C_r^\top D_r \\ D_r^\top C_r & D_r^\top D_r \end{bmatrix} + \begin{bmatrix} C_p^\top C_p & C_p^\top D_p \\ D_p^\top C_p & D_p^\top D_p \end{bmatrix} \preceq 0$$

- Is there a frequency domain inequality (FDI) equivalent to the LMI?
  - Not in general, but there is a necessary condition for the LMI to be feasible

Necessary condition: $\beta G_r(\bar{s})^\top G_r(s) - G_p(\bar{s})^\top G_p(s) \succeq 0$ for all $s \in \mathbb{C}$ with $s \notin \sigma(A)$, $\mathrm{Re}(s) \geq 0$.

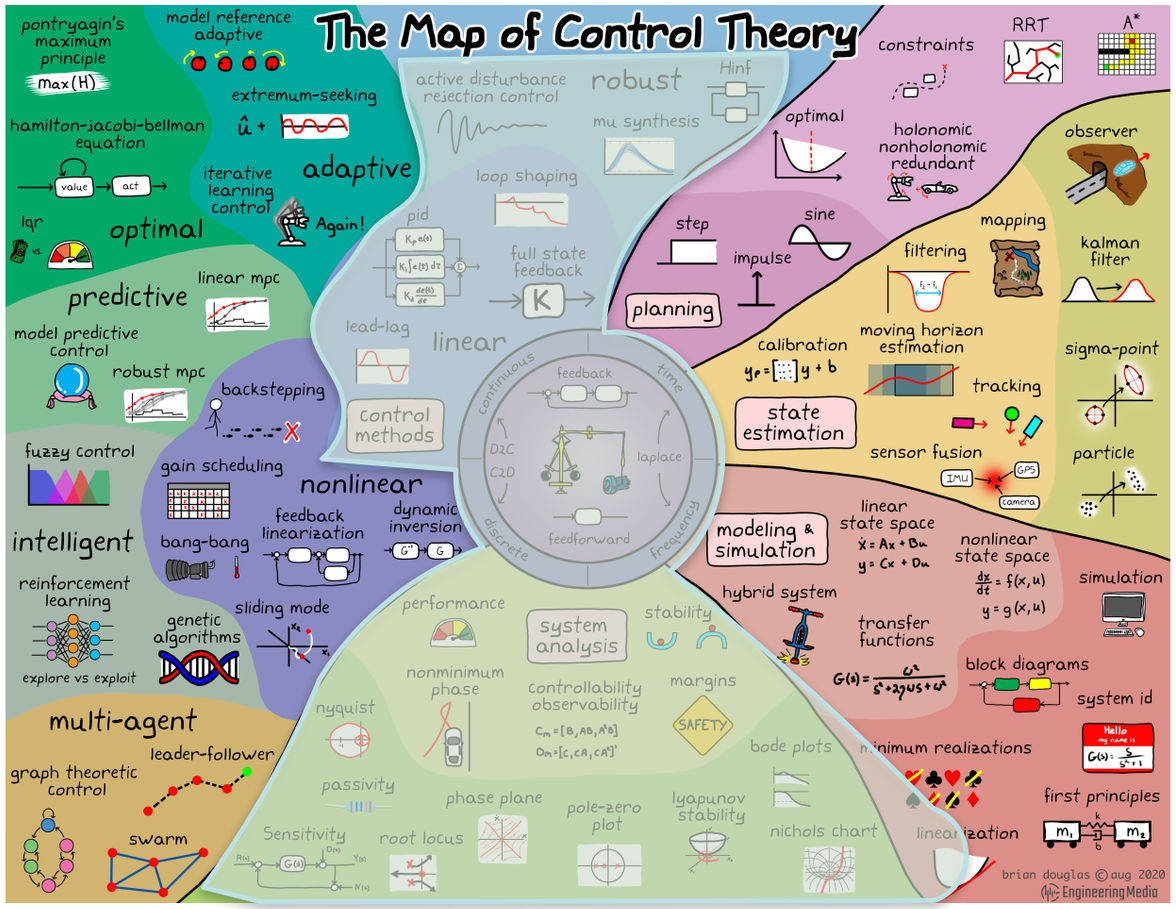
- Not sufficient, see a counter example in J. C. Willems, "On the existence of a nonpositive solution to the Riccati equation", IEEE Trans. Automat. Contr., vol. AC-19, pp. 592-593, Oct. 1974.
- But special cases do have an equivalent FDI:
  - see H_∞ norm (and the “Periodic Attacks” in later slides).
- Necessary condition relates to generalised singular values: $\mu(s) \triangleq \min_\beta \quad \sqrt{\beta}$

$$\text{s.t.} \quad \beta G_r(\bar{s})^\top G_r(s) - G_p(\bar{s})^\top G_p(s) \succeq 0$$

- For a single attack channel (input), we get: $\mu(s) = \dfrac{\overline{\sigma}_p(s)}{\underline{\sigma}_r(s)}$

$$\begin{aligned} \mu(s) \triangleq \min_\beta \quad & \sqrt{\beta} \\ \text{s.t.} \quad & \beta g_r(\bar{s})^\top g_r(s) - g_p(\bar{s})^\top g_p(s) \succeq 0 \\ = \min_\beta \quad & \sqrt{\beta} \\ \text{s.t.} \quad & \beta \underline{\sigma}_r(s)^2 - \overline{\sigma}_p(s)^2 \geq 0 \end{aligned}$$

Control System Design



Main steps in control system design:

- Scenario characterization
  - Models, Scenarios, Cost functions
- System Analysis
  - Sensitivity metrics (w.r.t. input)
  - **<u>Need to capture security aspects:</u>** maximise impact while undetected
- System Design (Control Methods)
  - Find control and monitoring algorithms that <u>minimise the sensitivity metric</u>
  - Re-design the structure of the system to <u>minimise the sensitivity metric</u>

Controller and Detector Design: Bilinear Matrix Inequalities

- **Design problem for** a Controller (L) and a Detector (K):
  - K and L change the matrices of the closed-loop system

$$\begin{aligned} \min_{K,L} \sup_{a \in \mathcal{L}_{2e}} \quad & \|y_p\|_{\mathcal{L}_2} \\ \text{s.t.} \quad & \|y_r\|_{\mathcal{L}_2} = 1 \\ & x(0) = 0 \end{aligned}$$

[Teixeira, Springer 2021]

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- **Designing** a Controller (L) & Detector (K) is possible under specific forms
  - … but leads to Bilinear Matrix Inequalities

[Teixeira, Springer 2021]

$$\min_{P \succeq 0, \beta > 0, K, L} \quad \beta$$

$$\text{s.t.} \quad \begin{bmatrix} A(K,L)^\top P + PA(K,L) & PB(K,L) & C_p(K,L)^\top \\ B(K,L)^\top P & 0 & D_p(K,L)^\top \\ C_p(K,L) & D_p(K,L) & \beta I \end{bmatrix} - \beta \begin{bmatrix} C_r \top \\ D_r^\top \\ 0 \end{bmatrix} \begin{bmatrix} C_r & D_r & 0 \end{bmatrix} \preceq 0,$$

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- Next we use an heuristic: alternating minimisation
  - Fix K,L, β, and solve for P
  - Fix P, solve for K,L, and β.

Example 1: Continuous-time

[Teixeira, Springer 2021]

- Classical vs Re-designed controller and detector

Nominal Design

After re-design

$$\mu(w) \triangleq \frac{\bar{\sigma}_p(jw)}{\underline{\sigma}_r(jw)}$$

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[Teixeira, Springer 2021]

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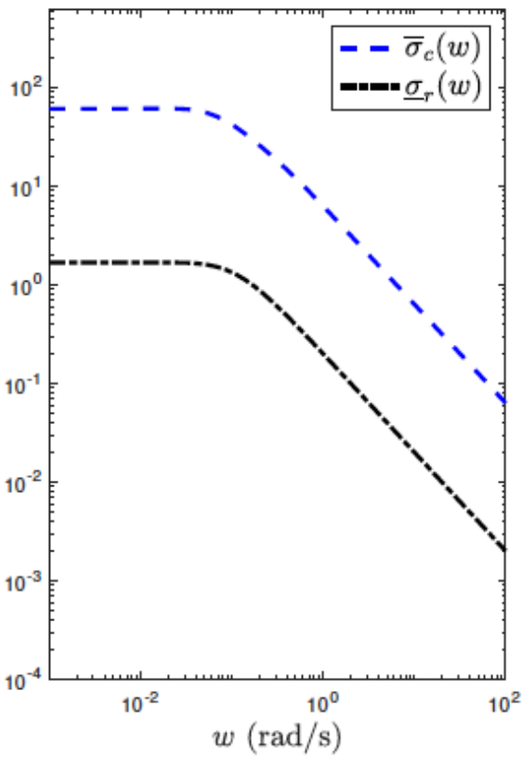

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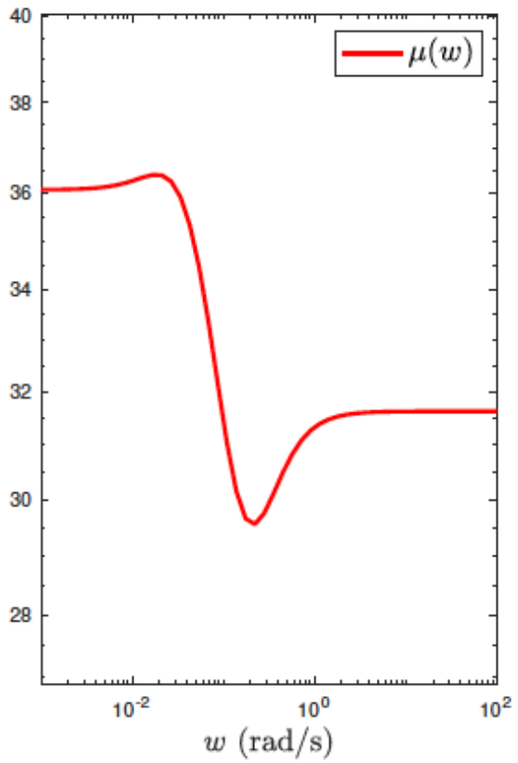

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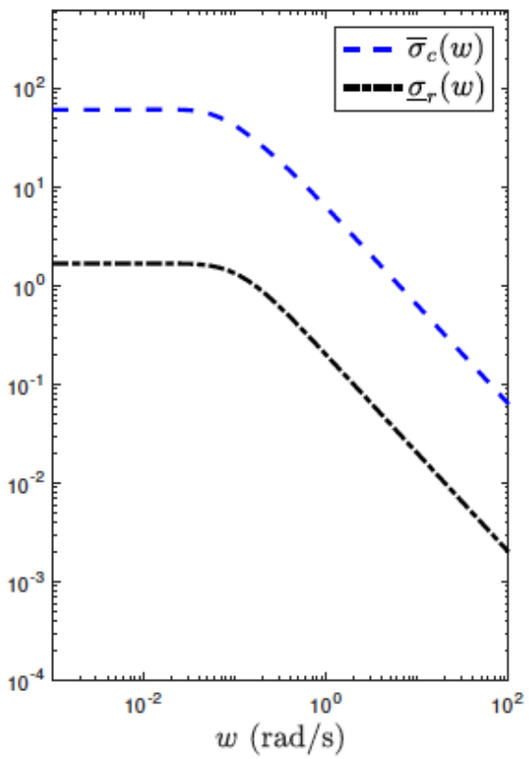

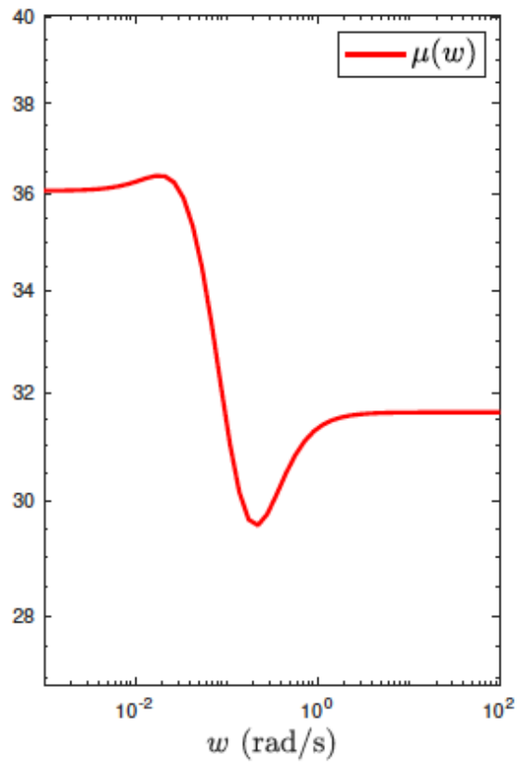

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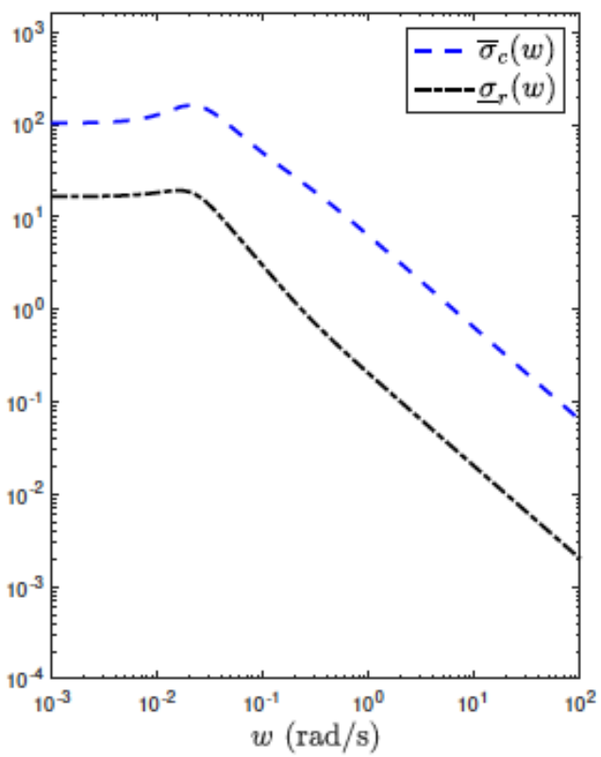

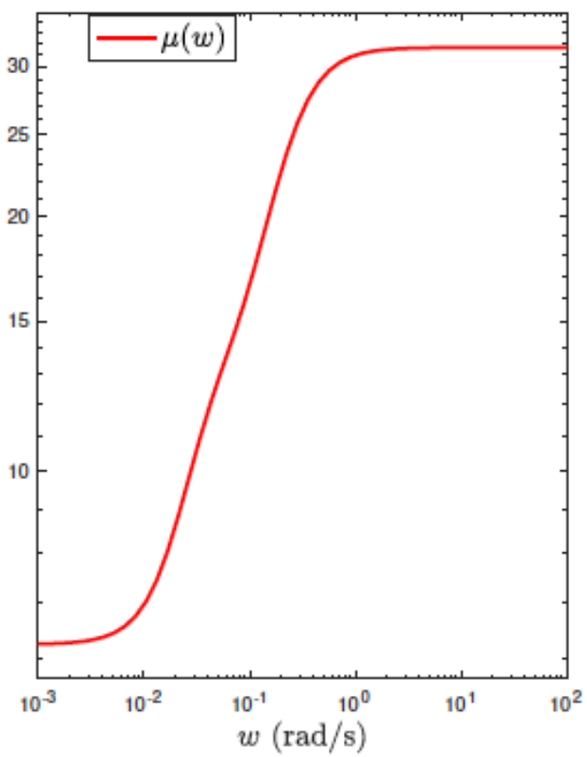

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Example 2: Discrete-time

[Anand and Teixeira, IFAC 2020]

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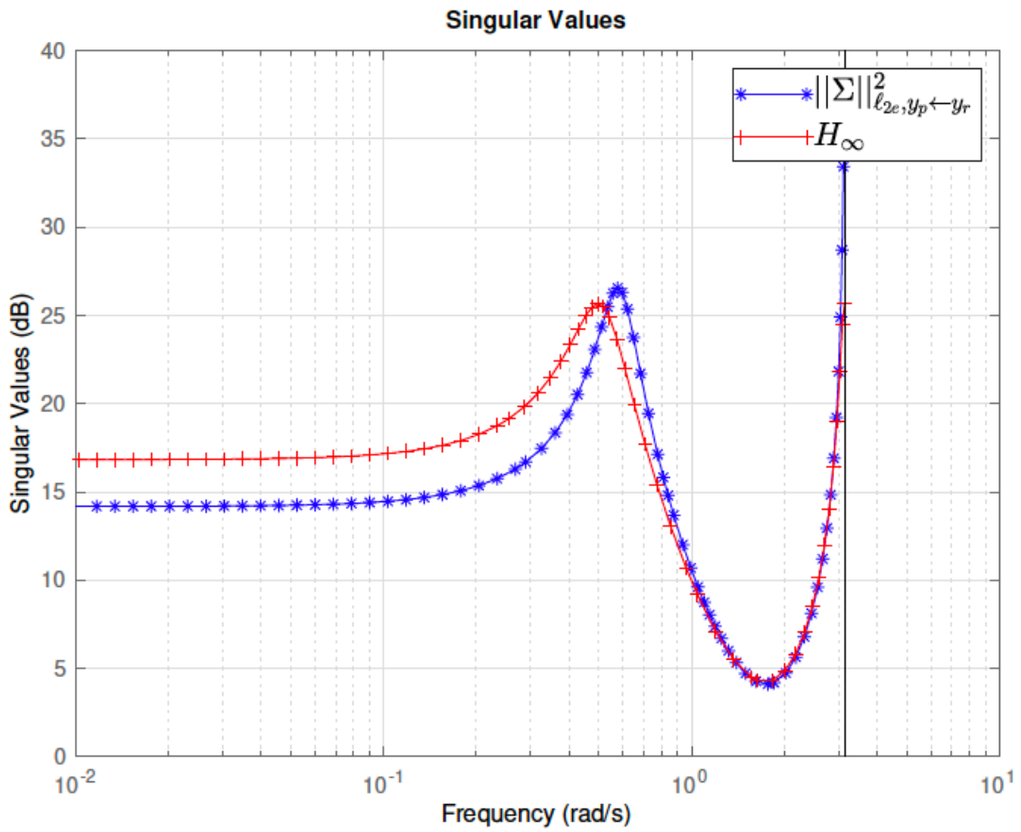

Fig. 1. Singular values - Performance output ($\bar{\sigma}(\Sigma_p)$)

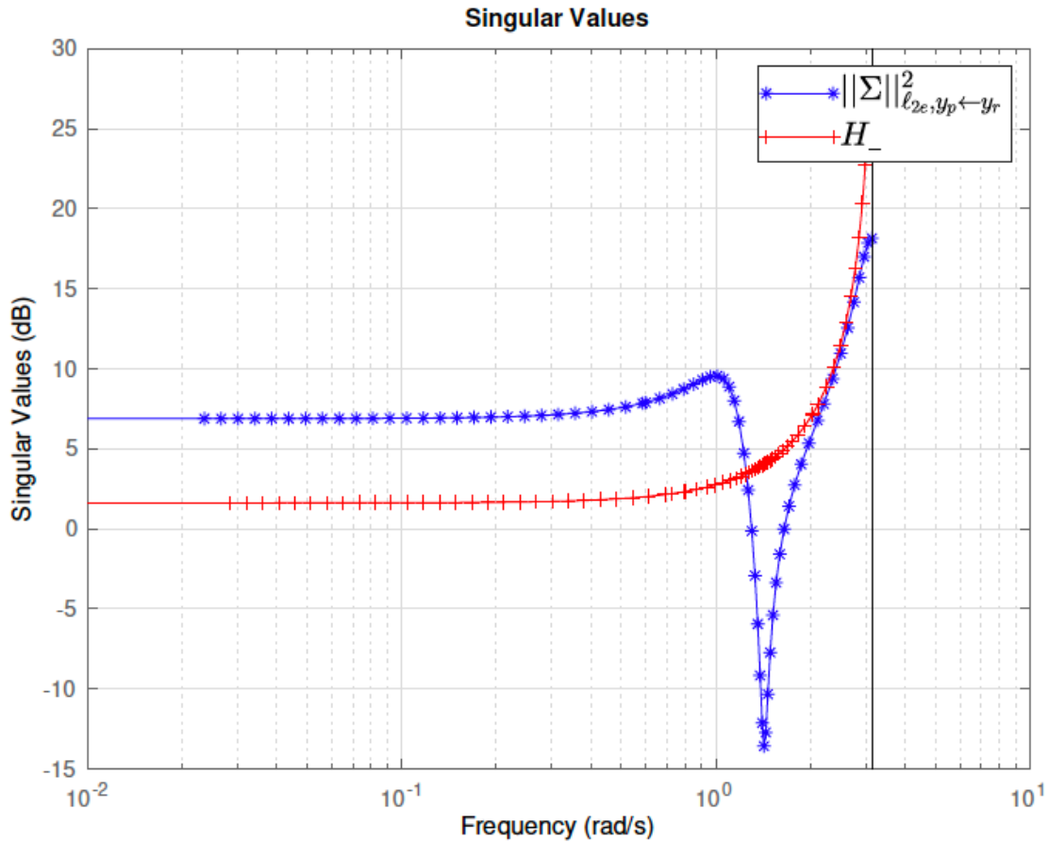

Fig. 2. Singular values - Detection output ($\underline{\sigma}(\Sigma_r)$)

UPPSALA UNIVERSITET

Security Metrics for Control Systems: from Analysis to Design

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- Can be cast as an SDP problem with BMIs

Structural Limitations

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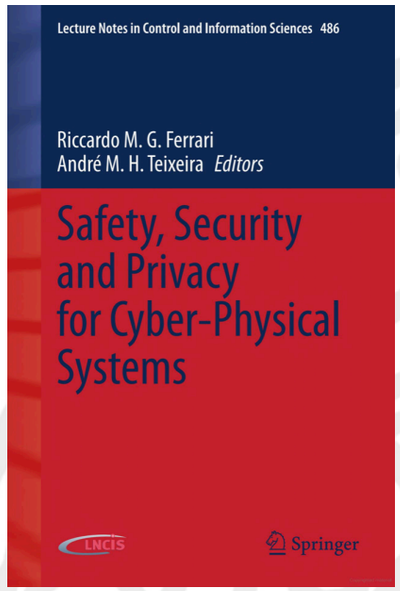
- **Relative degree:** "zeros at infinite frequencies"
  - relates to rate of decay of singular values at infinite frequency
  - Infeasible gain constraint if $\underline{\sigma}_r(jw)$ decays faster than $\overline{\sigma}_p(jw)$

Outline

- Cybersecurity in Control Systems
  - Security Game
  - Risk Management
    - Mapping to Control System Design
    - Example: undetectable attacks

- Security Metrics for Control Systems
  - Classical metrics in control engineering
  - Novel security metric for analysis and design
    - Analysis and design problems
    - Structural limitations - invariant zeros

- Additional topics
  - Variations of the adversary models and their respective metrics
  - Incorporating uncertainty for robust open-loop attacks



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- $\gamma^\star = \sup_{\omega} \ \mu(w)$ (see slide 31 on FDI)
  - For single input case*: $\mu(w) \triangleq \dfrac{\bar{\sigma}_p(jw)}{\underline{\sigma}_r(jw)}$

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- Truncated attacks

$$\sup_{a \in \mathcal{L}_{2e}} \|y_p\|_{\mathcal{L}_2} \quad \text{s.t.} \quad \|y_r\|_{\mathcal{L}_2} = 1, \; x(-\infty) = 0, \; x(\infty) = 0, \; a(t) = 0, \; \forall t \geq 0$$

[Teixeira, CDC 19]

* part of Tutorial 1

Structural limitation: a closer look

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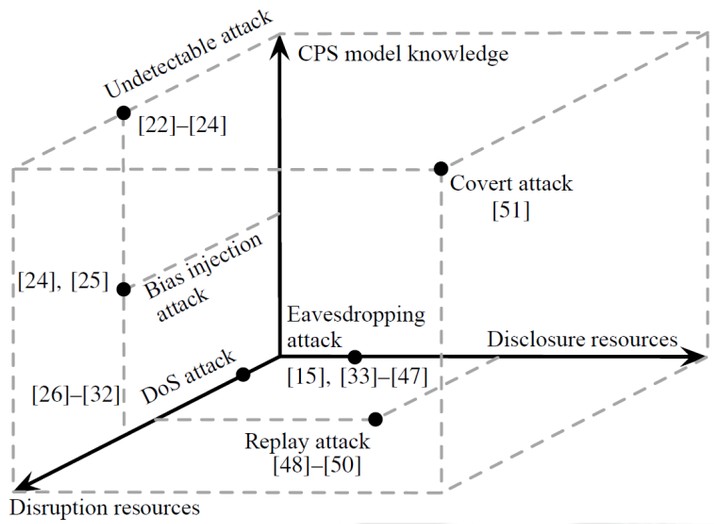

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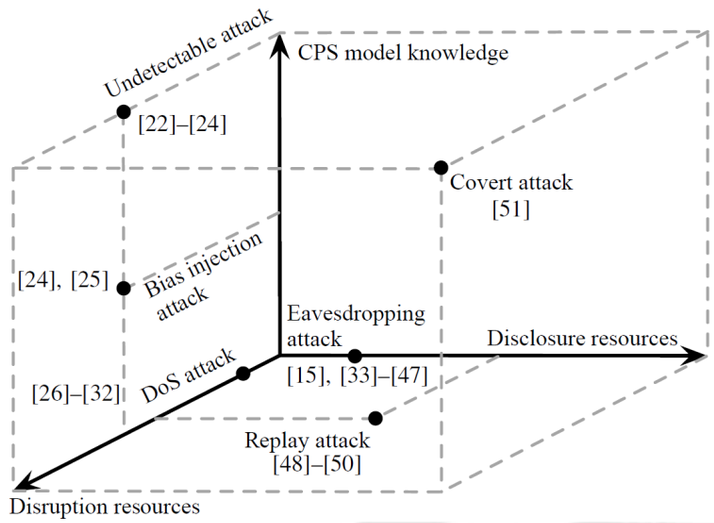
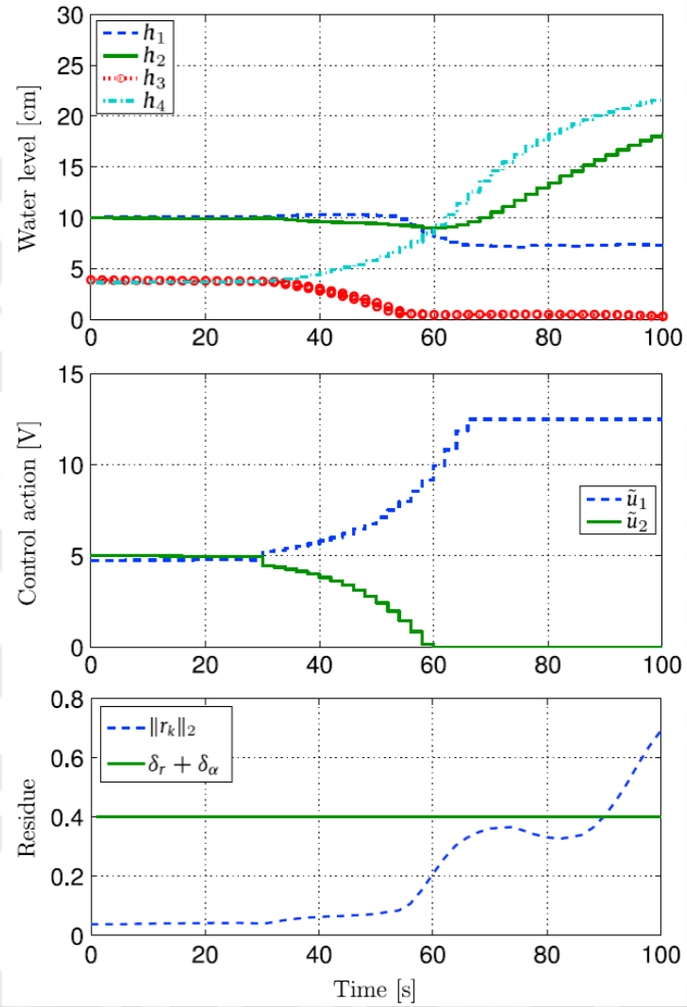


- Undetectable attacks need accurate models
  - Deviations make the attacks detectable
  - Basic principle for “Watermarking & Moving Target” defense - obfuscate part of the model



Structural limitation: a closer look

- Fundamental structural limitations still exist
  - Optimal attacks can give unbounded impact while undetected, <u>regardless</u> of the controller and detector
  - Related to undetectable attacks

- Undetectable attacks need accurate models
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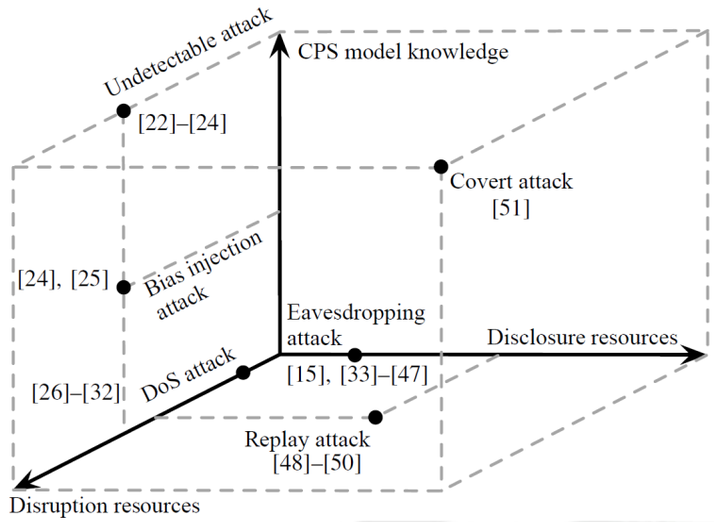
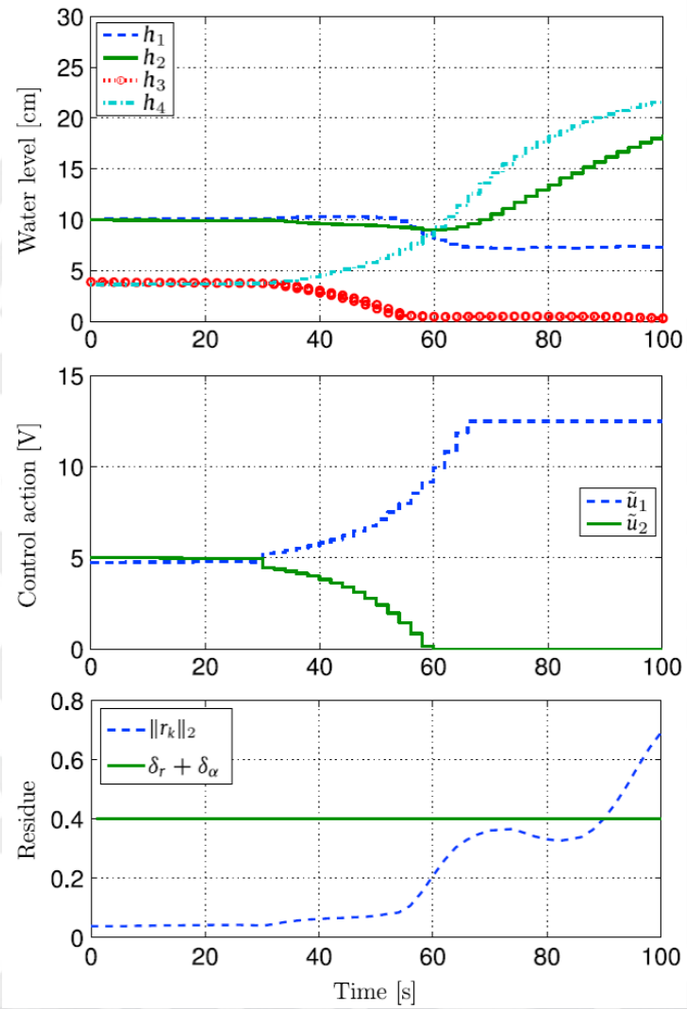
- Can we capture this element in the security metrics?





Uncertainty & Robust Attacks

[Anand, under review]

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- System with static parameter uncertainty

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\begin{aligned}
x[k+1] &= A_{cl}^{\Delta} x[k] + B_{cl}^{\Delta} a[k] \\
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- Sampled approximation

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Example

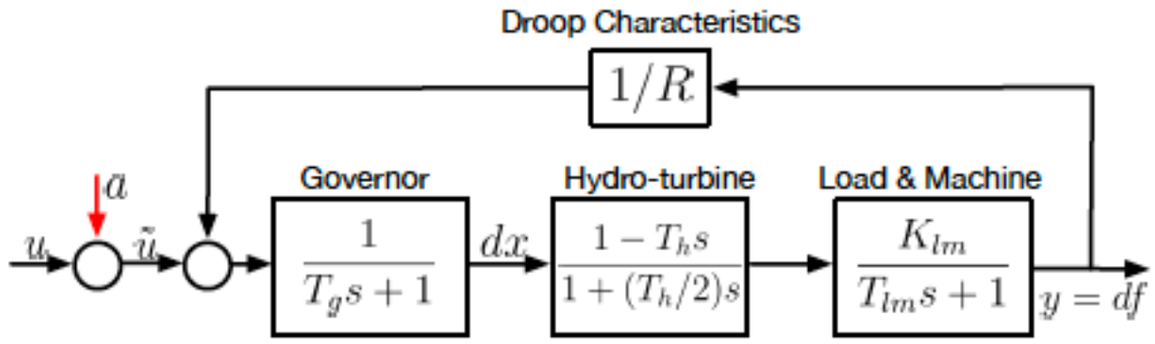

Fig. 4. Power generating system with a hydro turbine.

SYSTEM PARAMETERS

K_{lm}	1	T_{lm}	6
T_g	0.2	R	0.05
T_h	[4 6]	T_s	0.1

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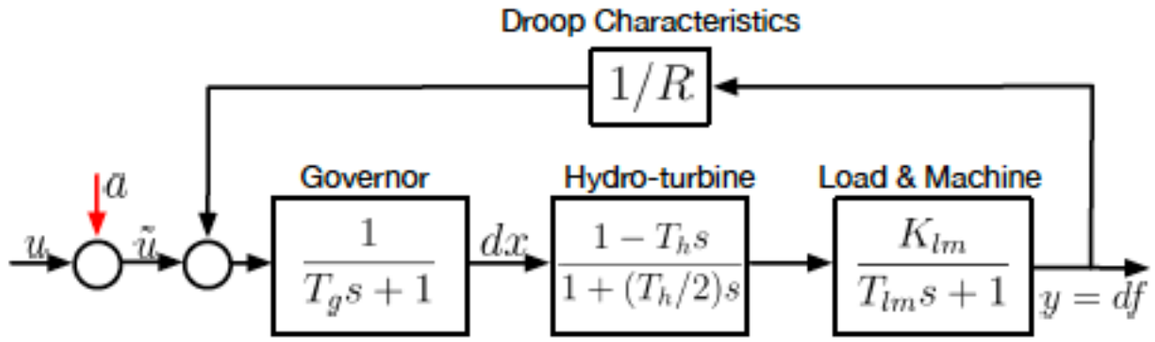

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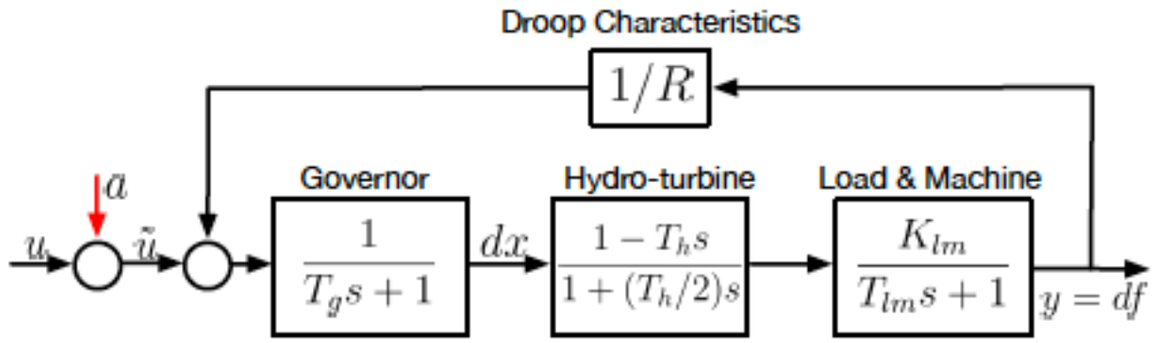

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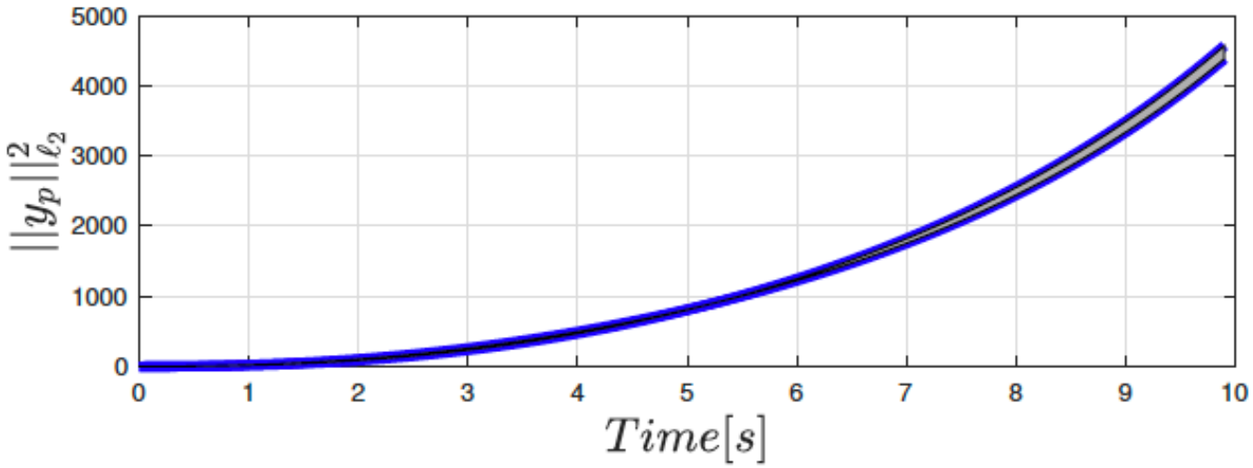

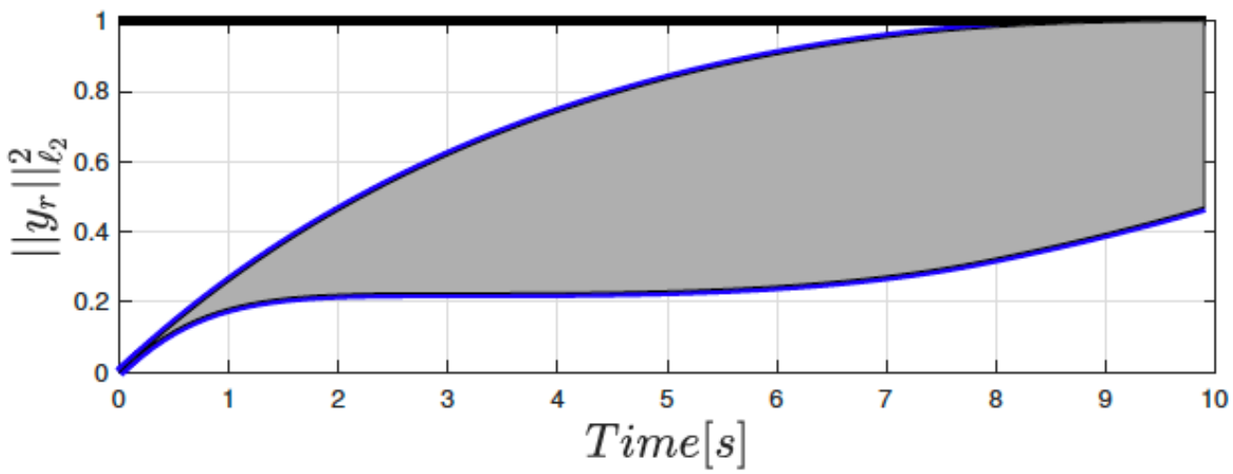

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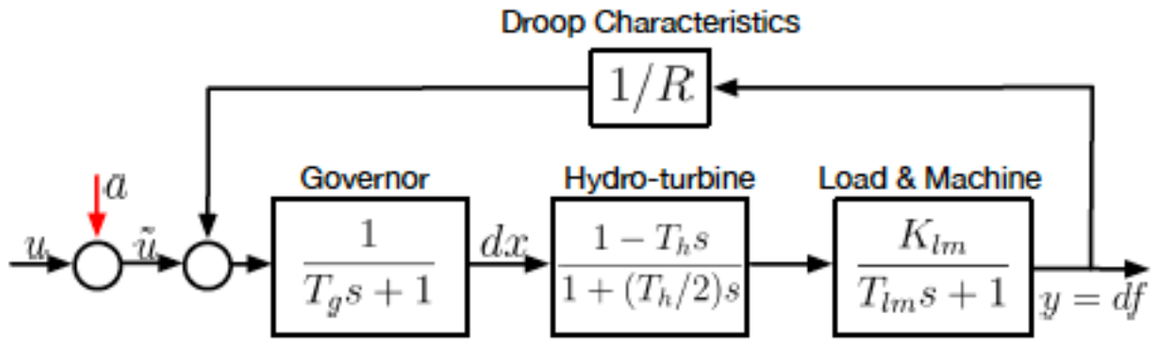

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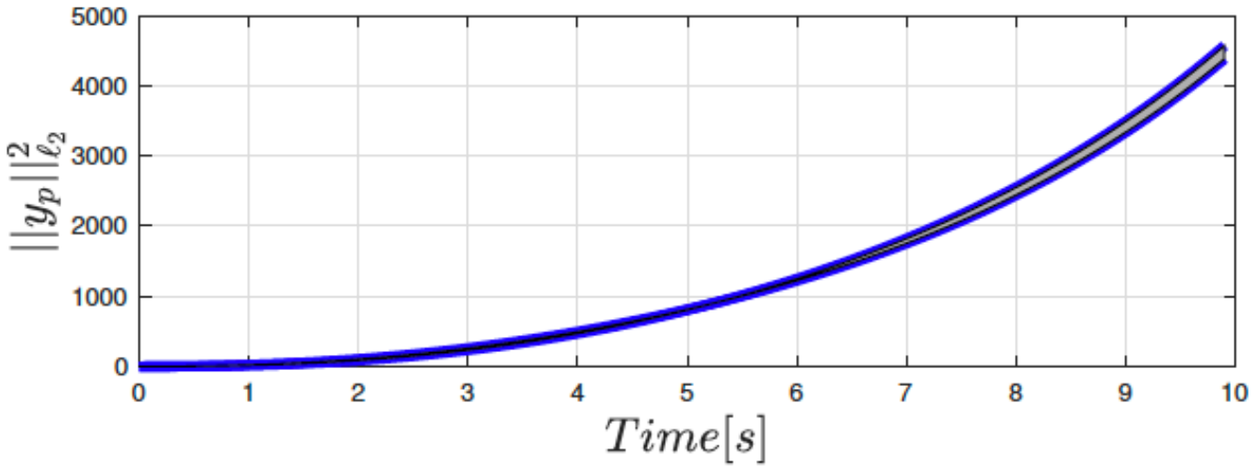
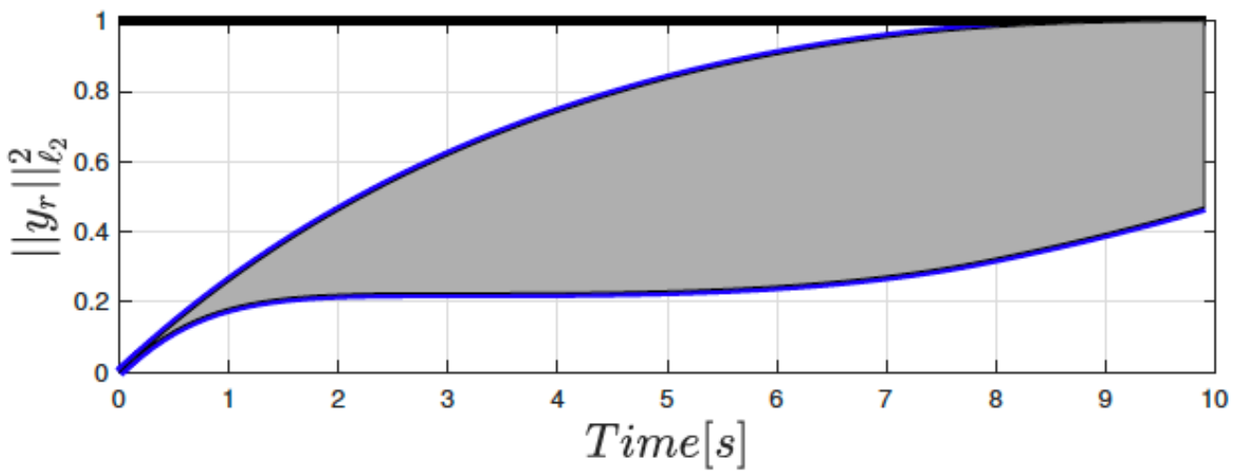
- "Uncertainty as a defense" can be incorporated by design
  - Watermarking, moving target, weak encryption, …





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Summary

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  - Example: undetectable attacks

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- Novel security metrics enables new defense mechanisms
  - Controller / Detector **design** through BMIs
  - “Uncertainty as defense”