Risk Management and Game Theory for Securing Control Systems

The Subtle Interplays between Adversary Models, Security Risk Metrics, and Uncertainty

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Swedish Research Council

Swedish Foundation for Strategic Research

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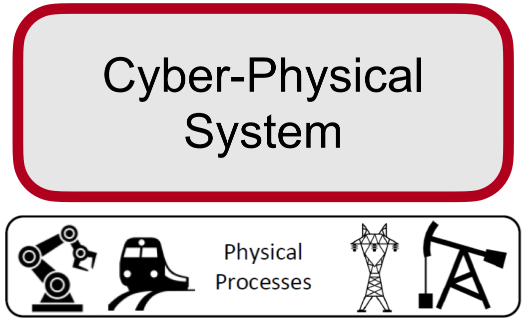
Uppsala Universitet

Outline

- Security Risk Management
- Scenario and Threat Models
- Security Metrics and Game-Theoretic Design
- Security under Model Uncertainty
- Probabilistic Risk Measures and Game-Theoretic Design
- Conclusions and Remarks

“The Security Game”: key ingredients



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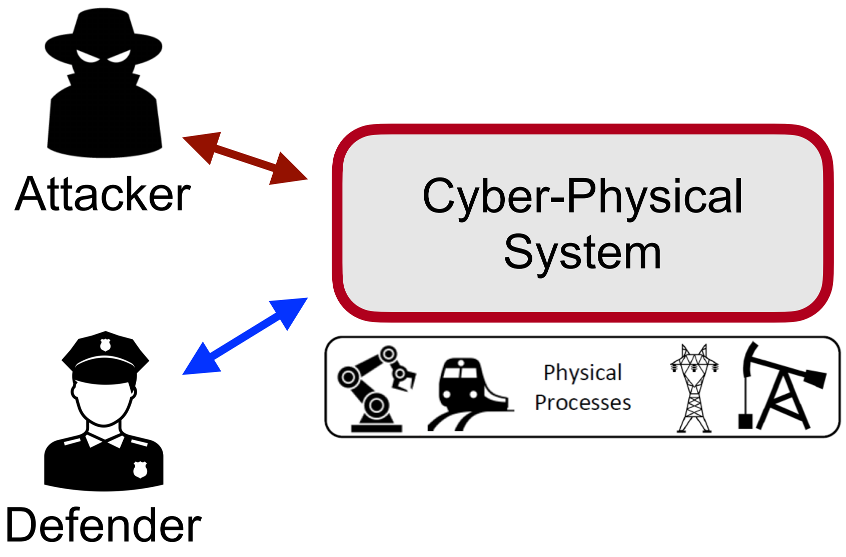

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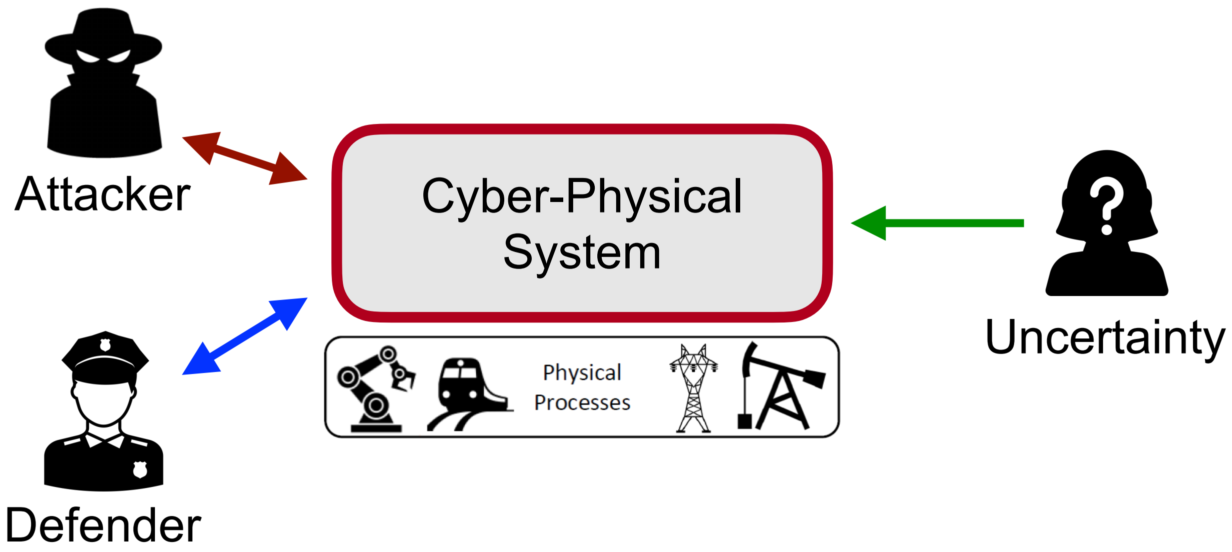

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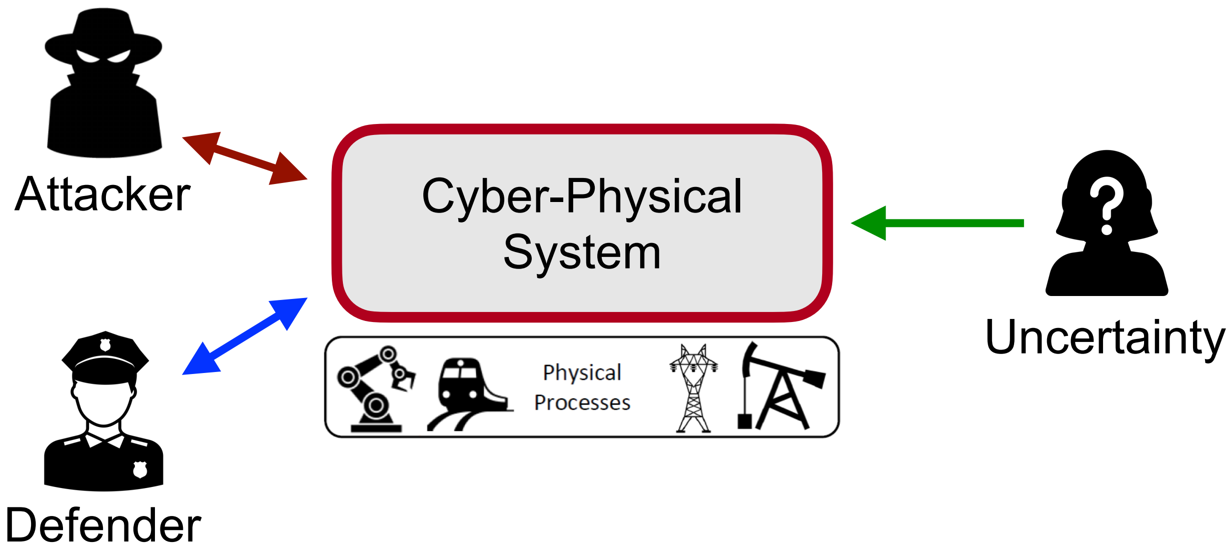

Need tools and strategies to understand and mitigate attacks:

- **Which threats** should we care about?
- **What impact** can we expect from attacks?
- **Which resources** should we **protect**, and how?

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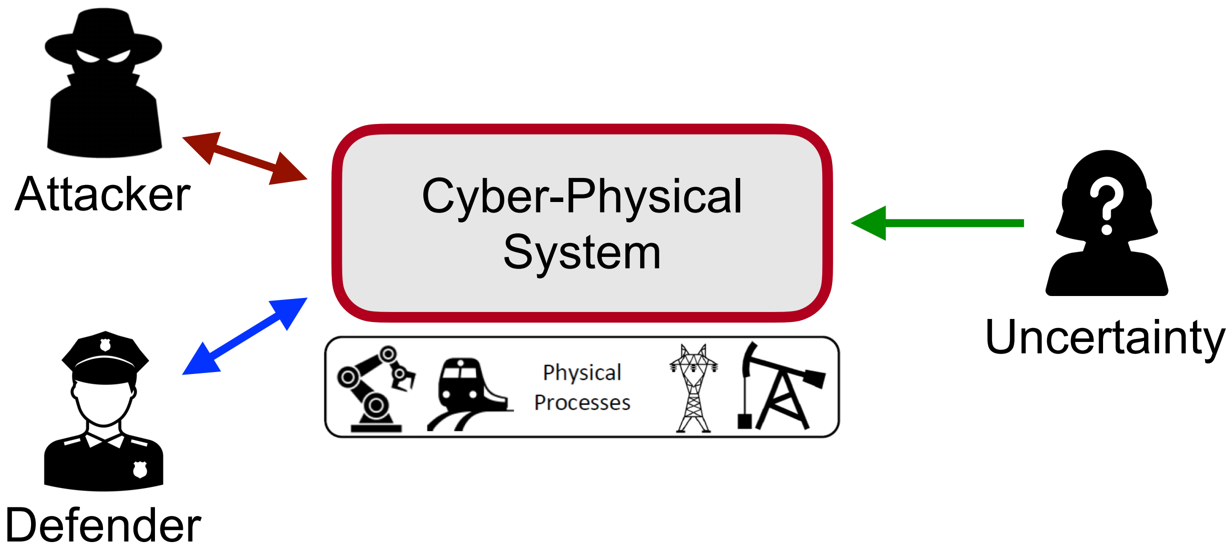

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- **Which threats** should we care about?
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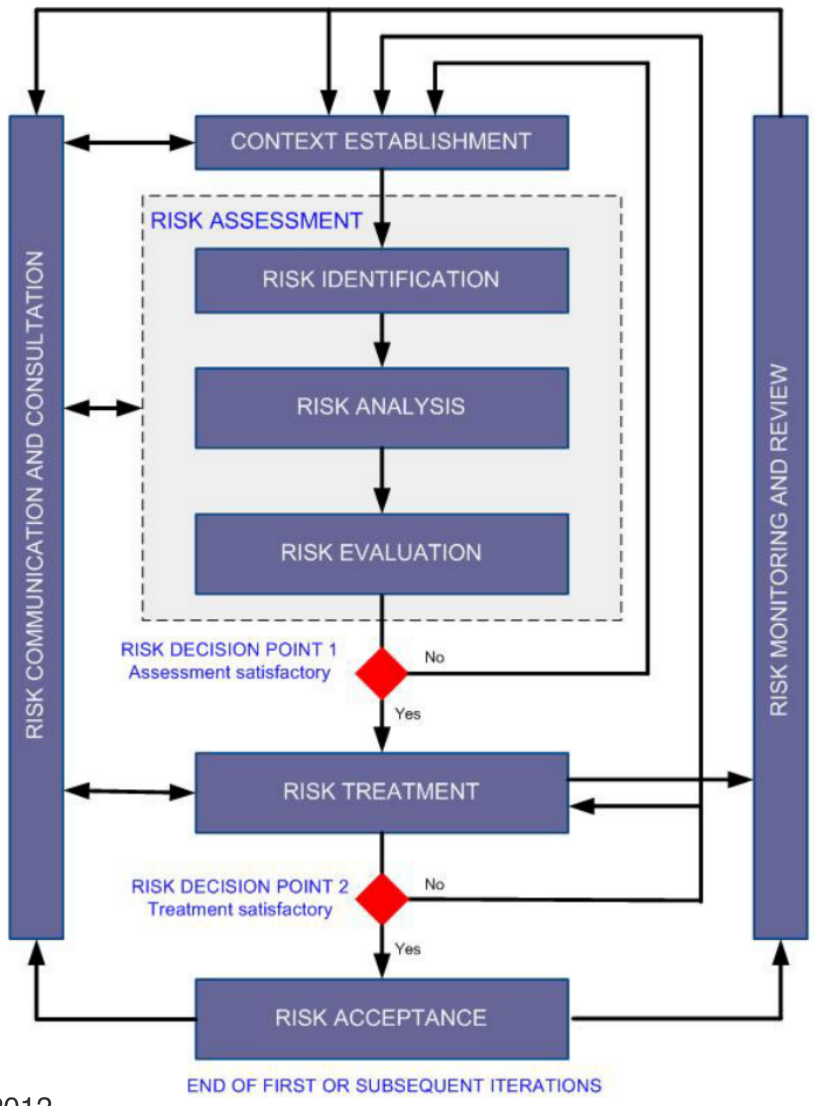
- How to find answers: **Risk Management & Game Theory + Control Theory + Statistical Learning**

Risk Management Cycle

Risk = (Scenario, Likelihood, Impact)

[Kaplan & Garrick, 1981]



[ISO 31000]

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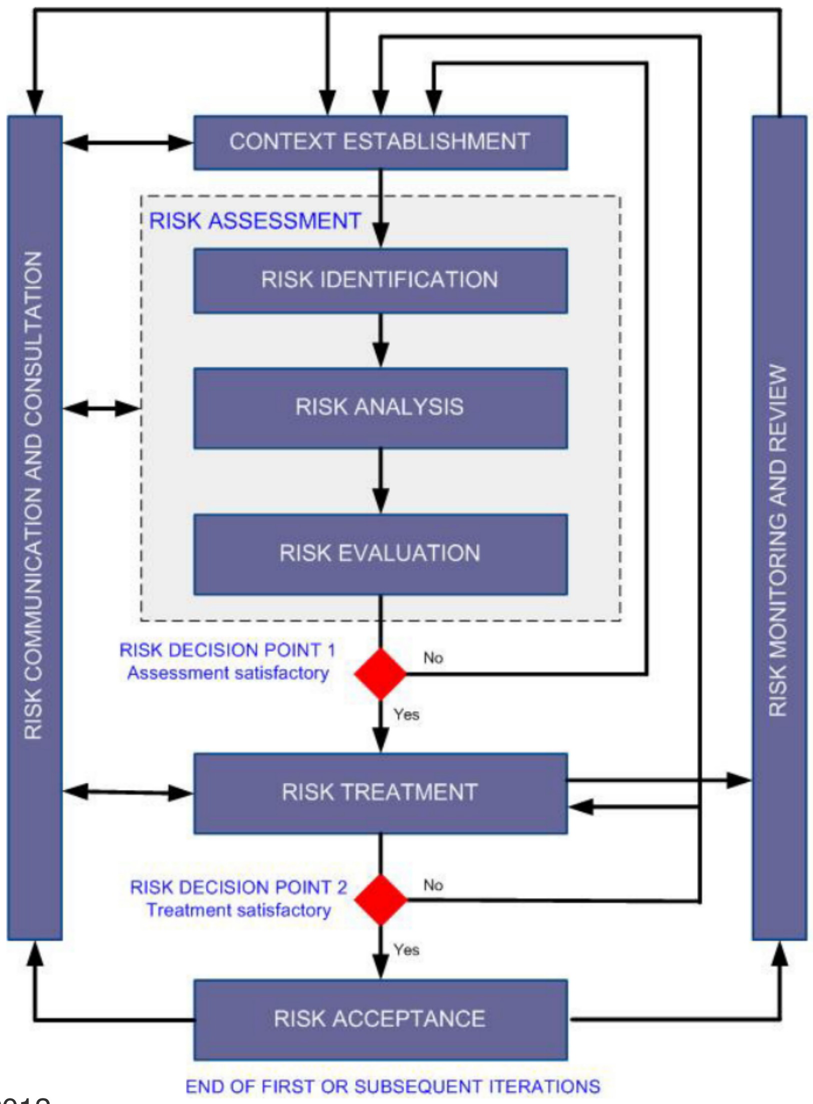
Main steps in risk management



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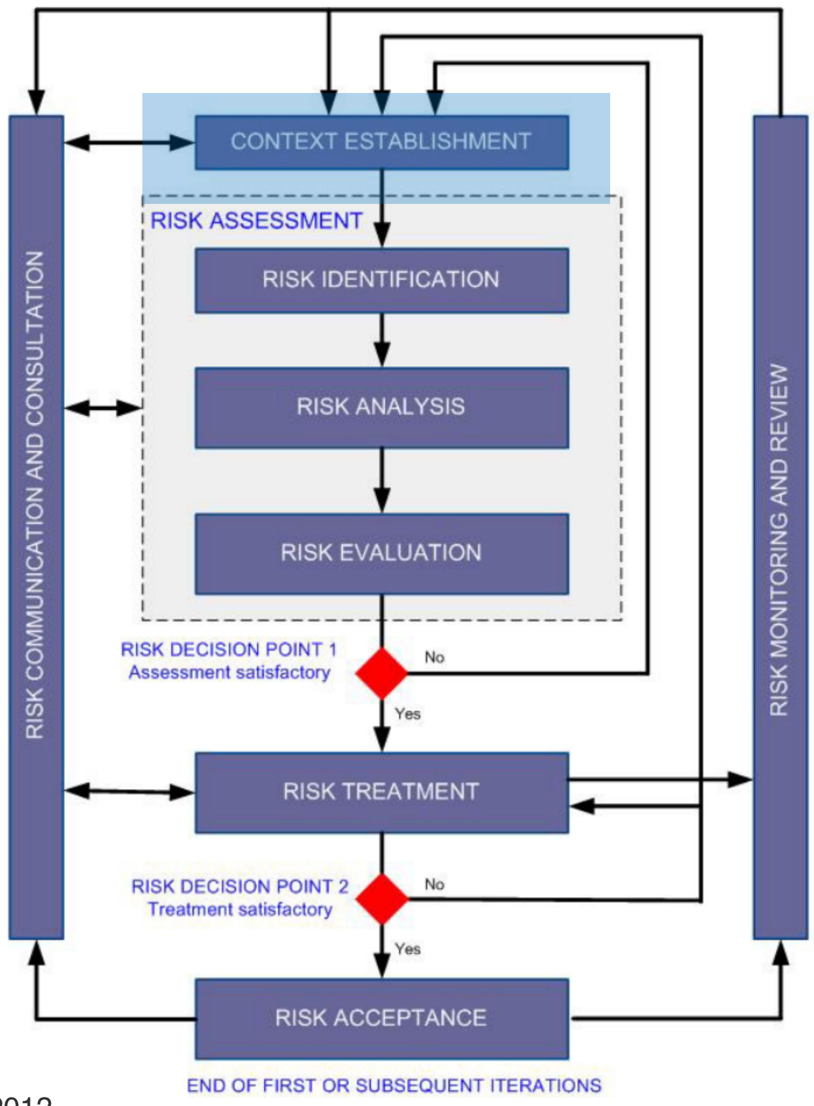

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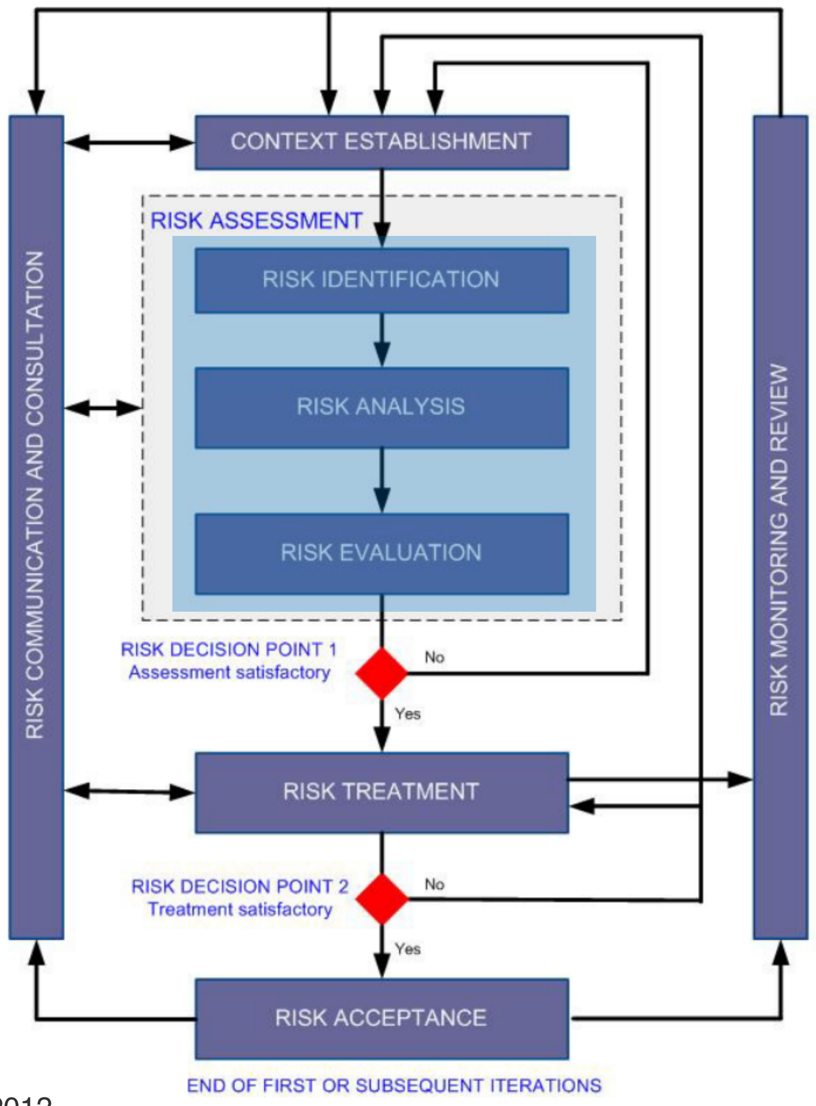

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- Risk Analysis
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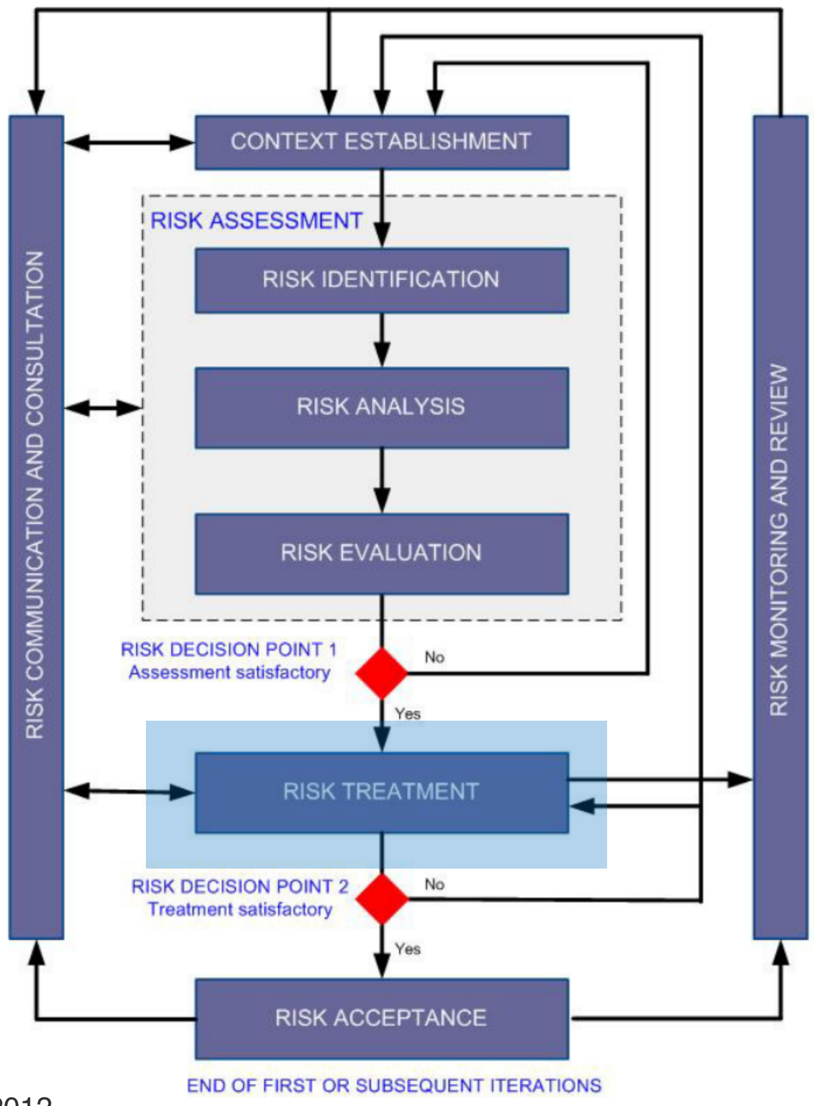
- Risk Mitigation
    - Prevention, Detection, Treatment



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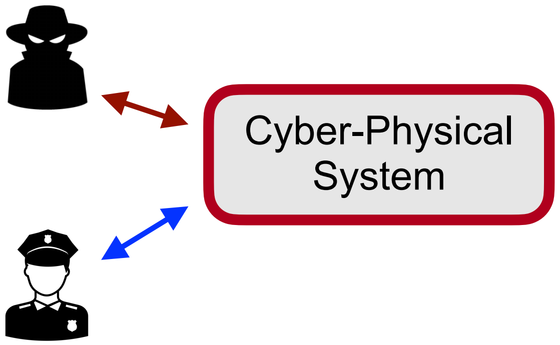

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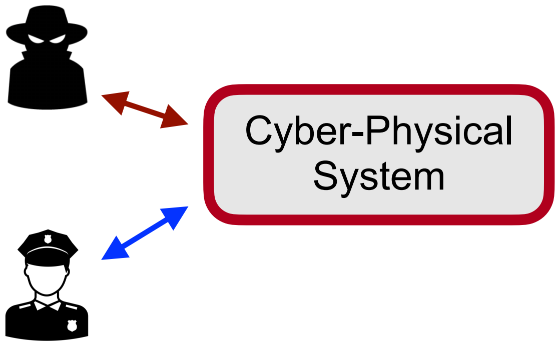

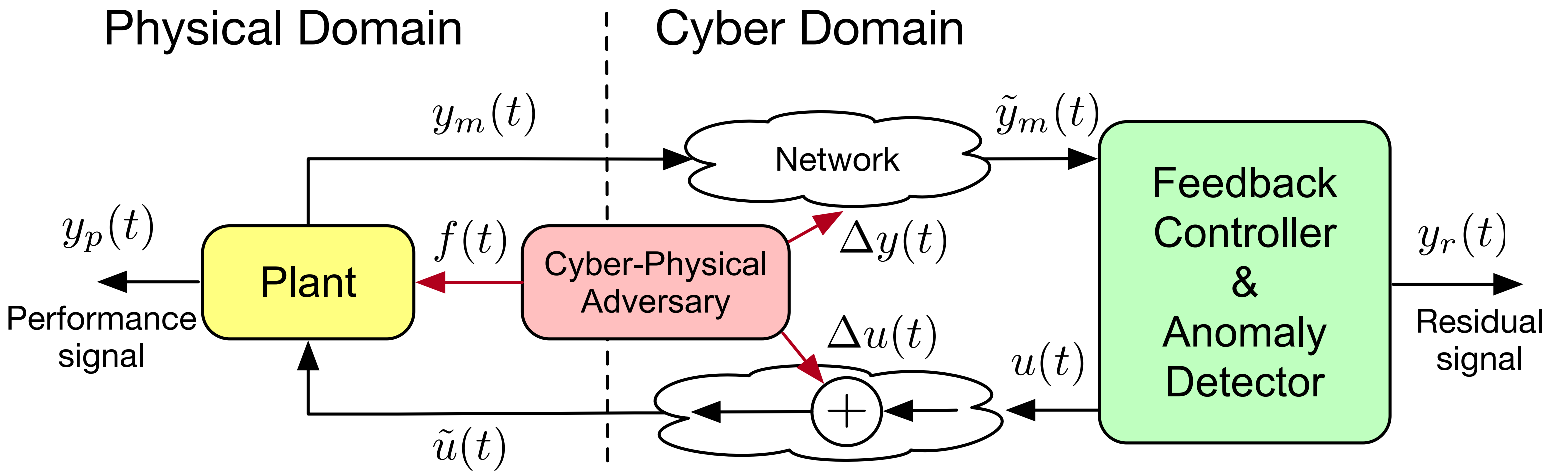

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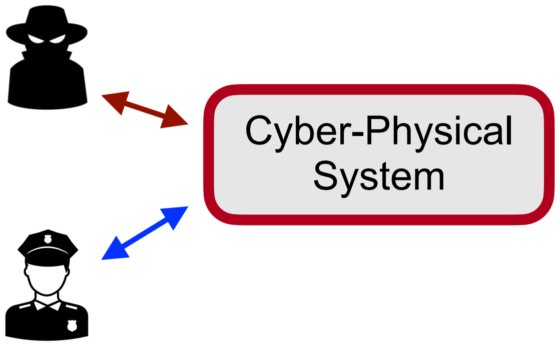

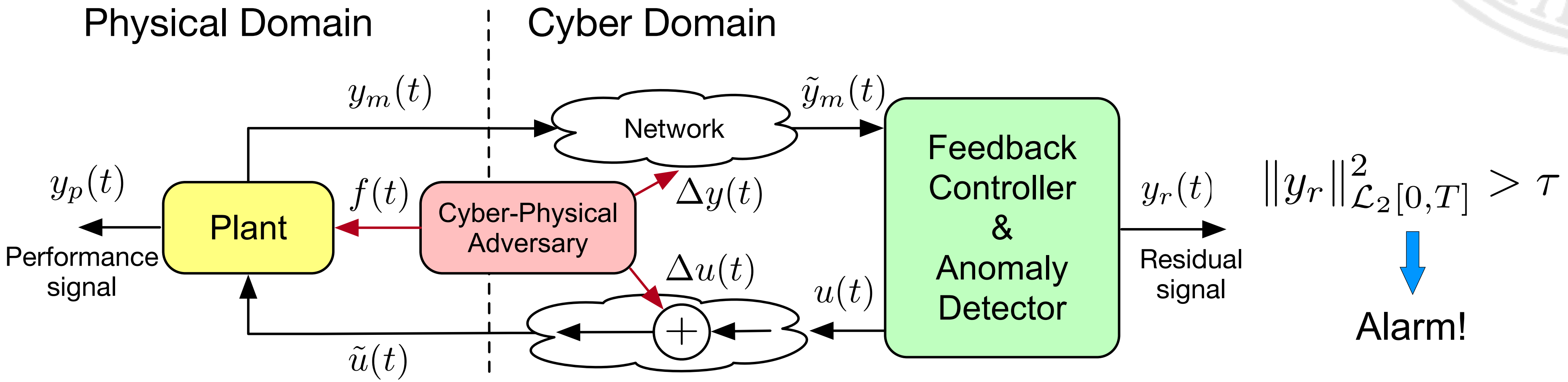

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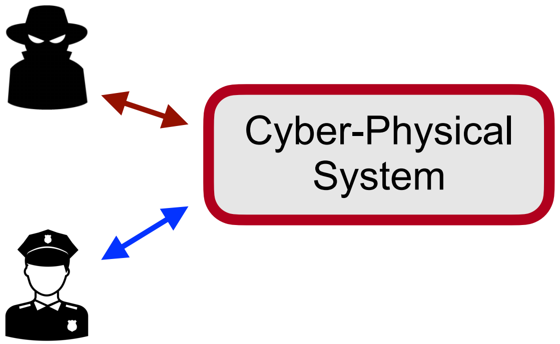

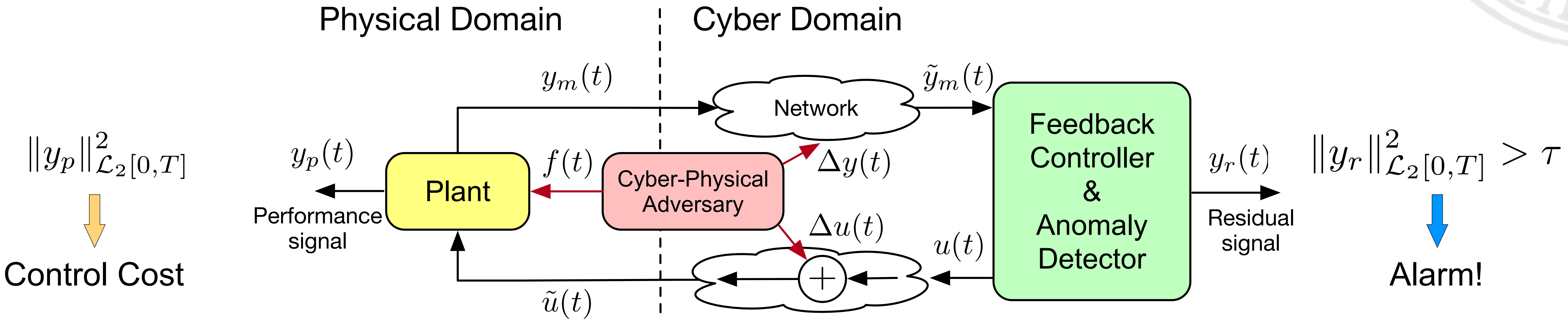

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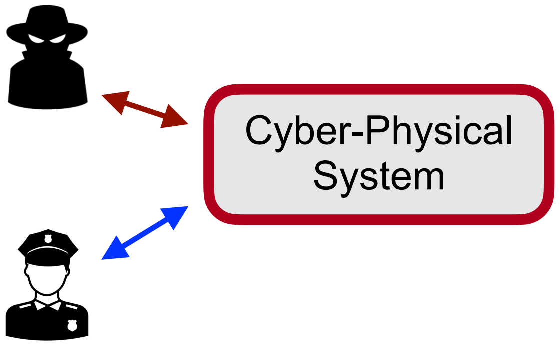

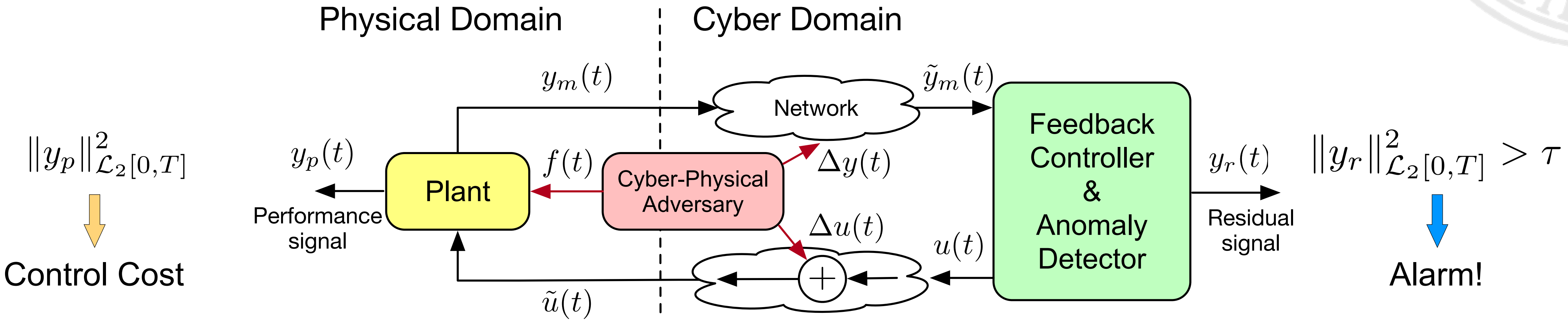

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$a(t)$

Attack signal

$y_r(t)$

Closed-loop system

Adversary Models

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Key elements [Do 2019]
- Goals
- Assumptions
- Capabilities

Do et al., "The role of the adversary model in applied security research", Computers & Security, 2019

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They define *what* the system is (in)secure against.

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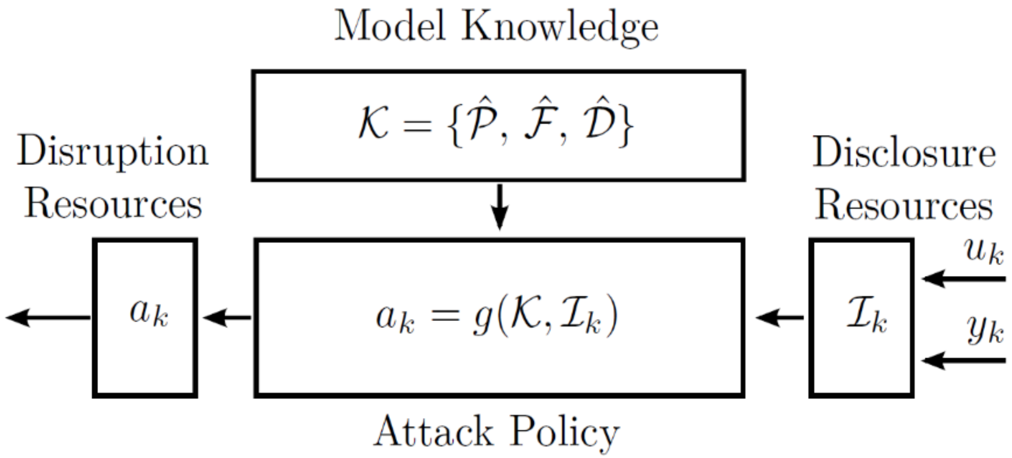

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They define *what* the system is (in)secure against.

- **Attack policy:** Goal of the attack? Destroy equipment, increase costs, *remain undetected*…
- **CPS model knowledge:** Adversary knows models of plant and controller? Better models increase possibility for stealthy attacks…
- **Disruption/disclosure resources:** Which channels can the adversary access?

[Teixeira *et al*., Automatica, 2015]

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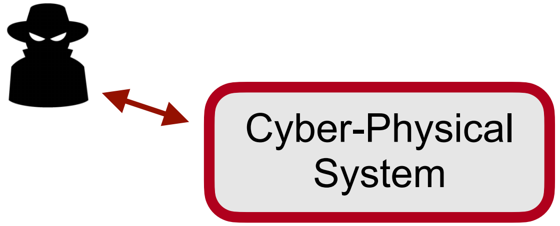
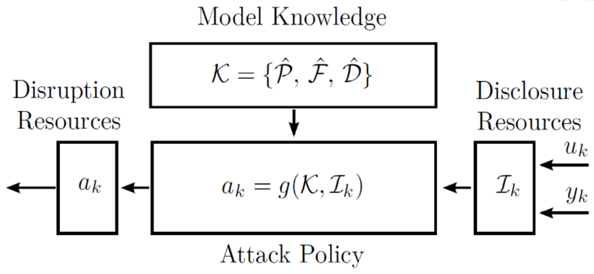
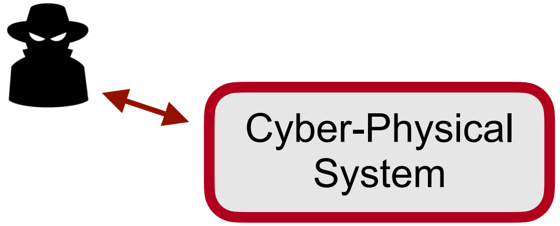
Attack Scenarios







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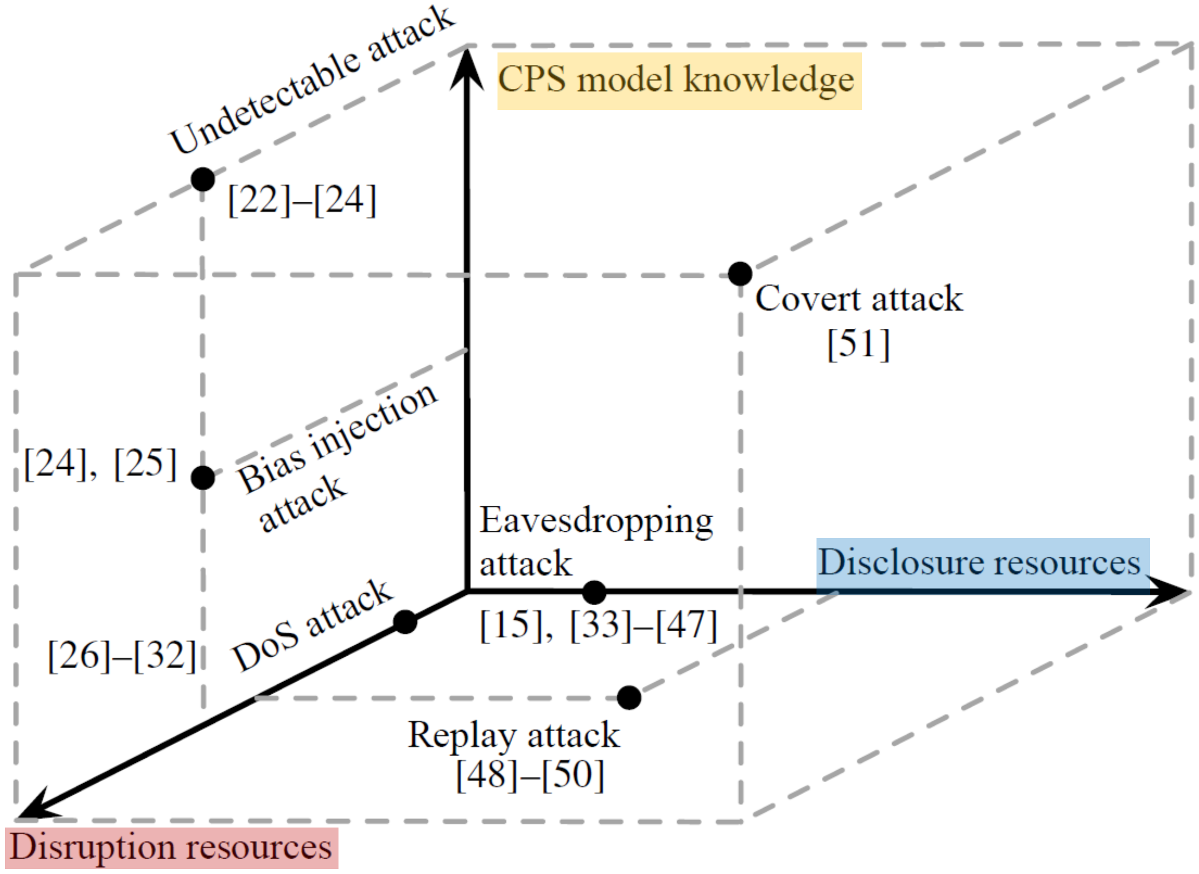

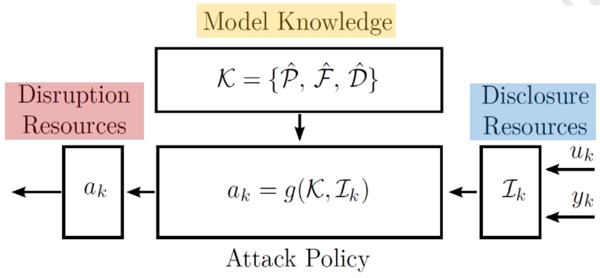

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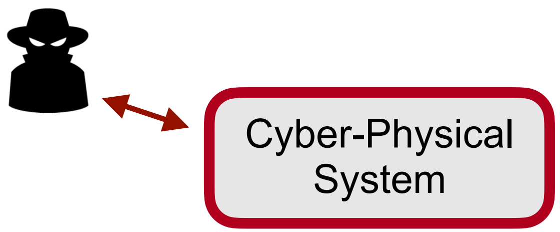

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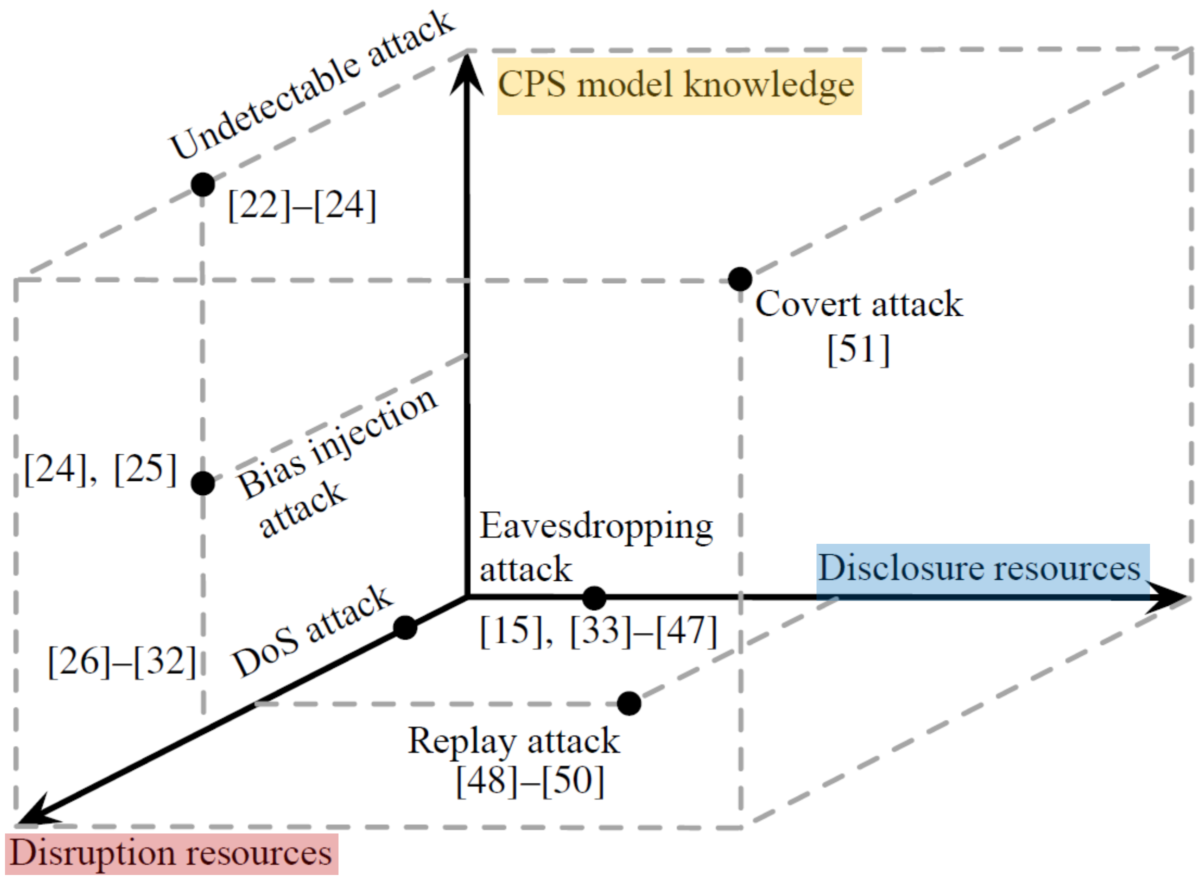
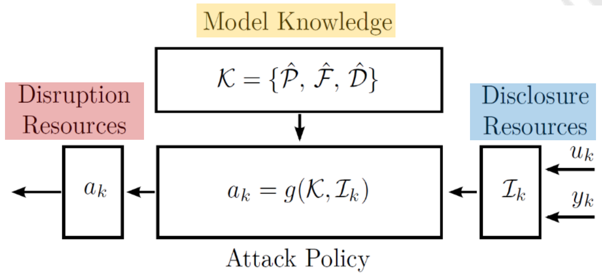




Adversary Models

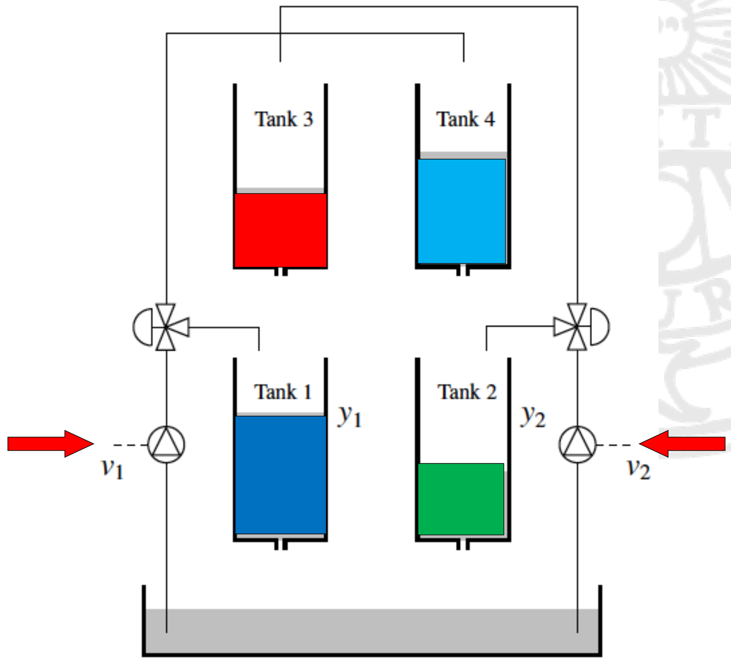
- How does the adversary behave against the system?

Security Analysis:

- Can it evade detection?
- Can it violate safety?
- How complex/likely is it?

[Chong *et al.*, ECC, 2019] [Teixeira *et al.*, Automatica, 2015]

Example: Zero Dynamics Attack



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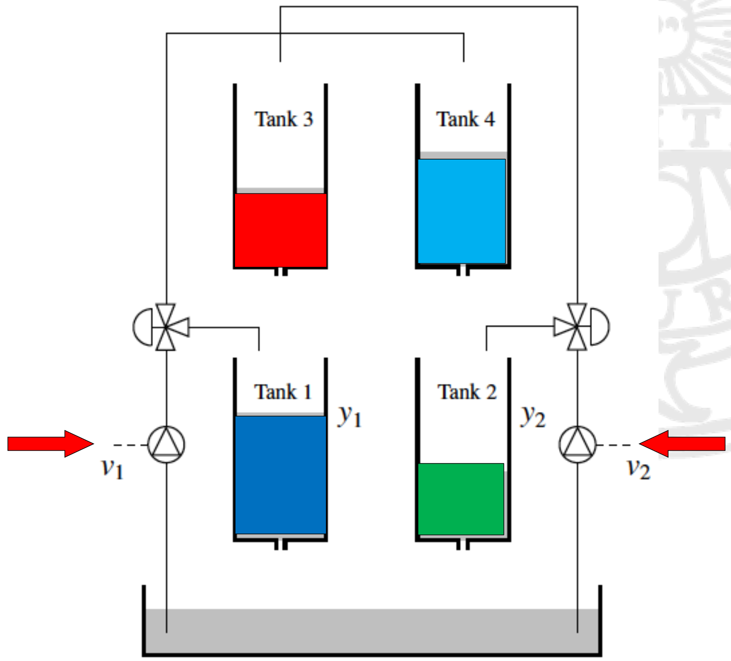
Closed-loop system



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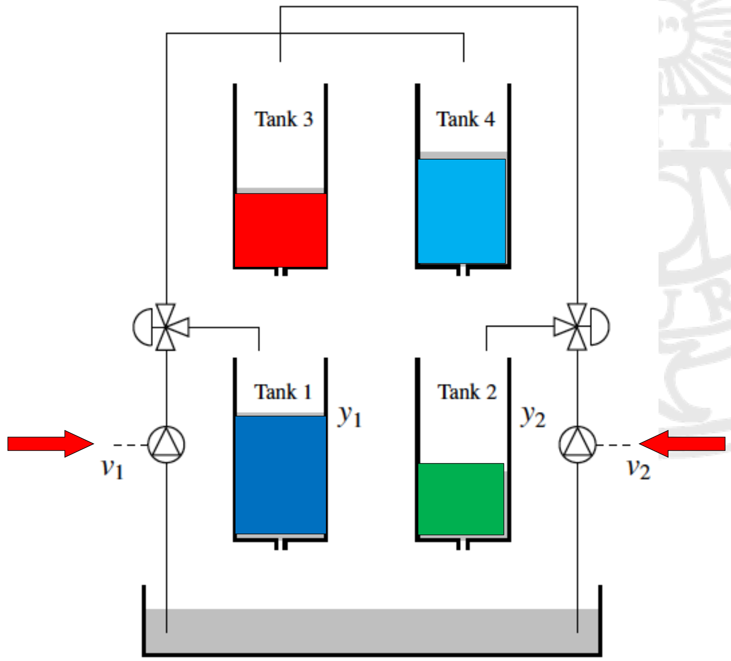

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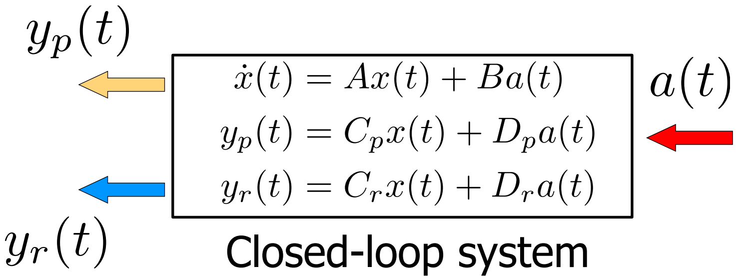

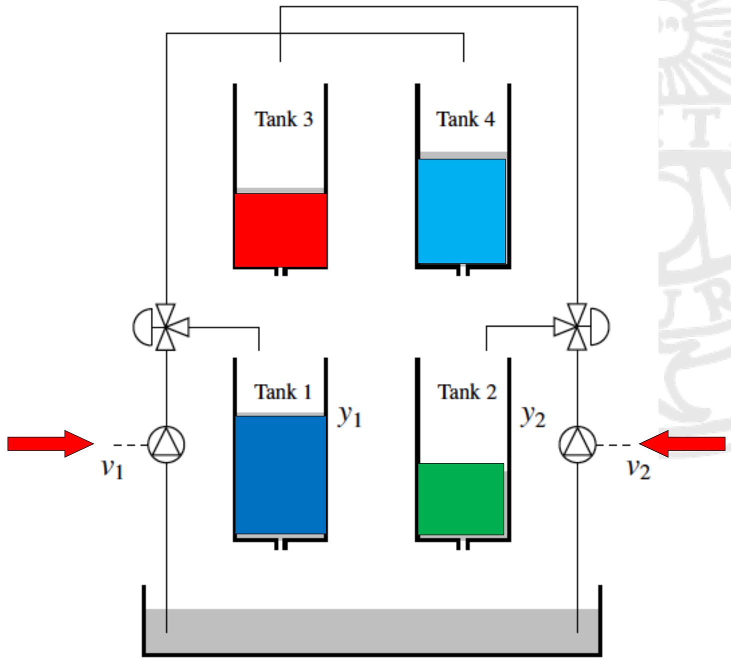

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  - $|\nu| < 1$: vanishing attack
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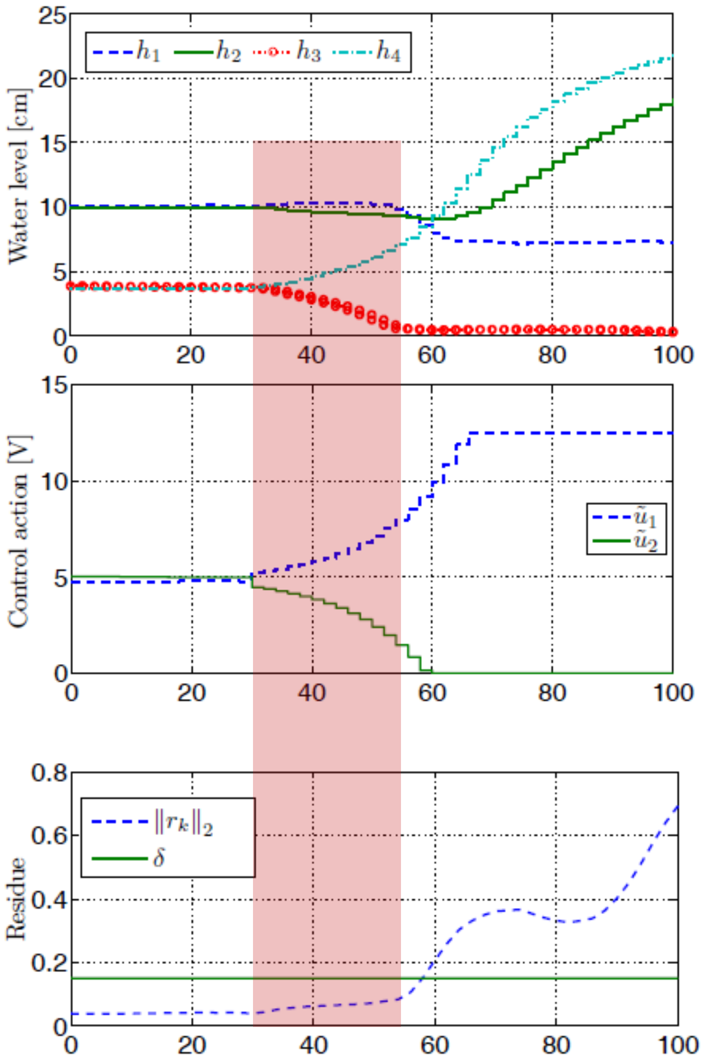

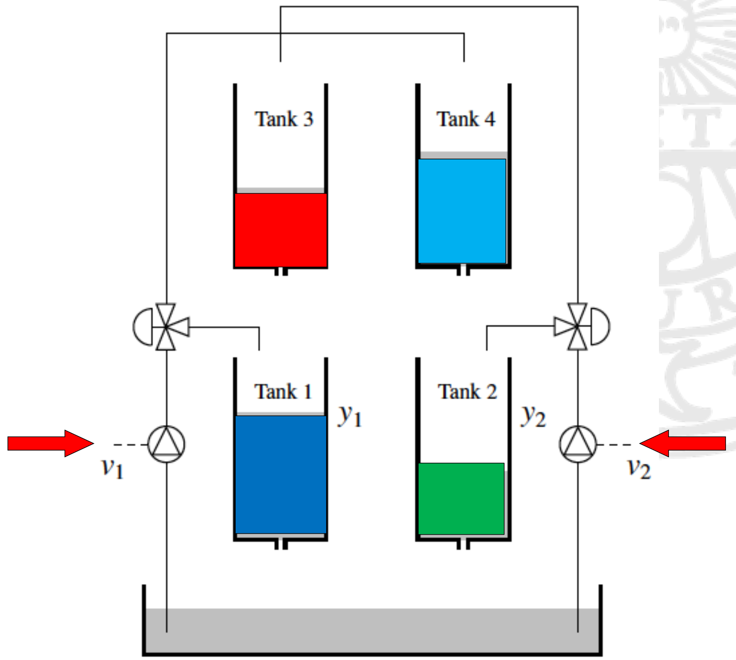

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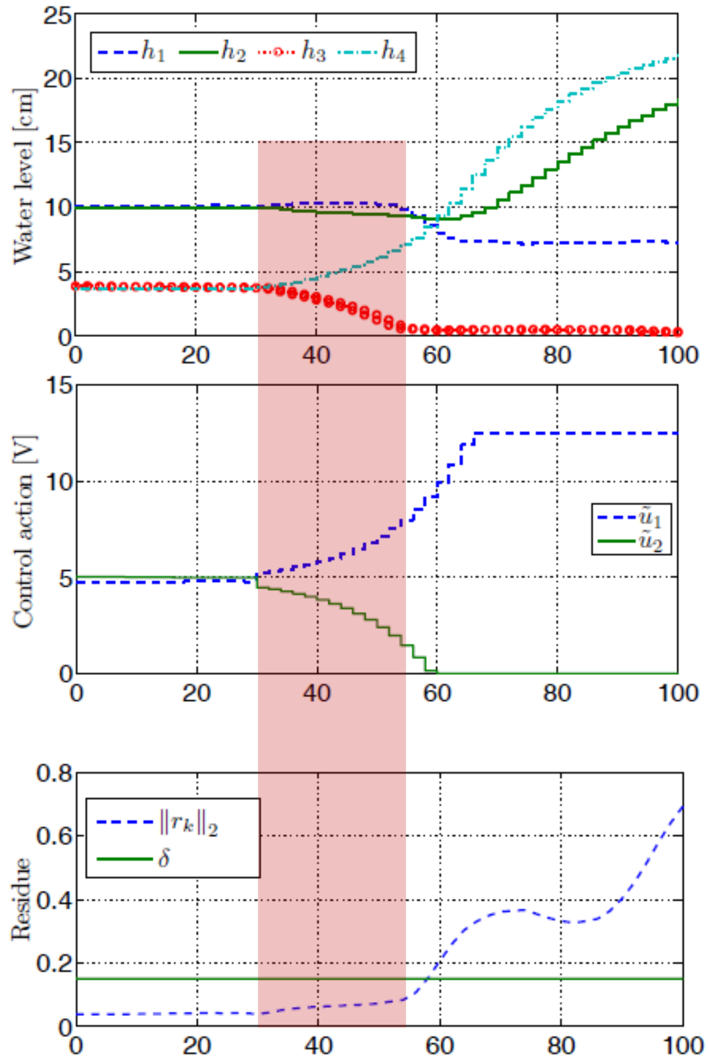

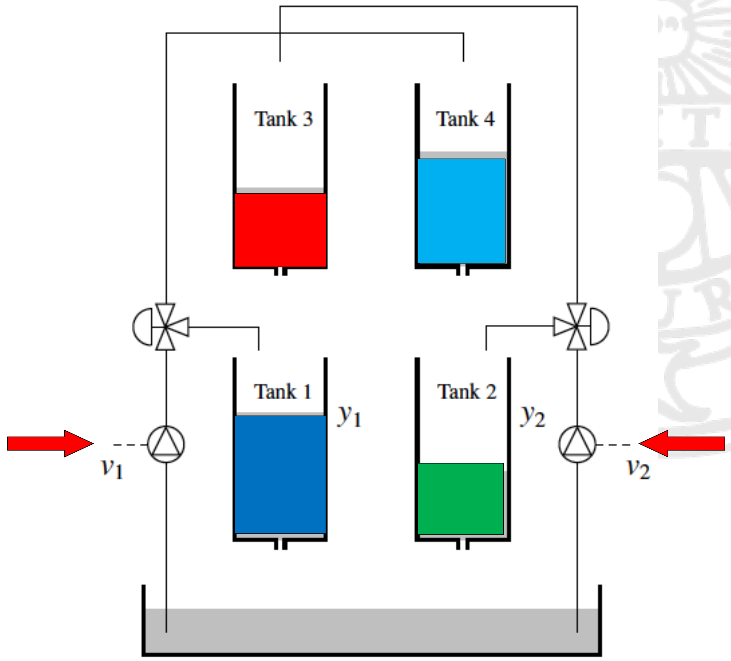

Attack is undetected during the "linear" regime.

Attack impact is significant: empties Tank 3.

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Defense: Active Detection of Attacks

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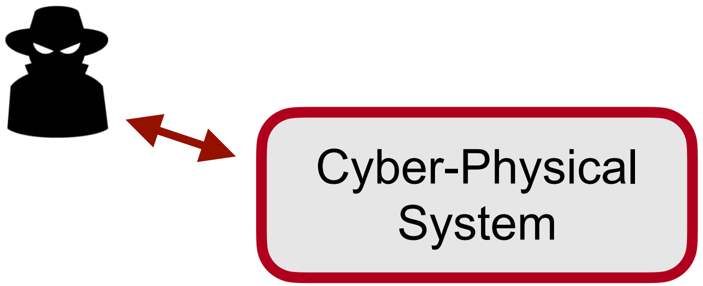

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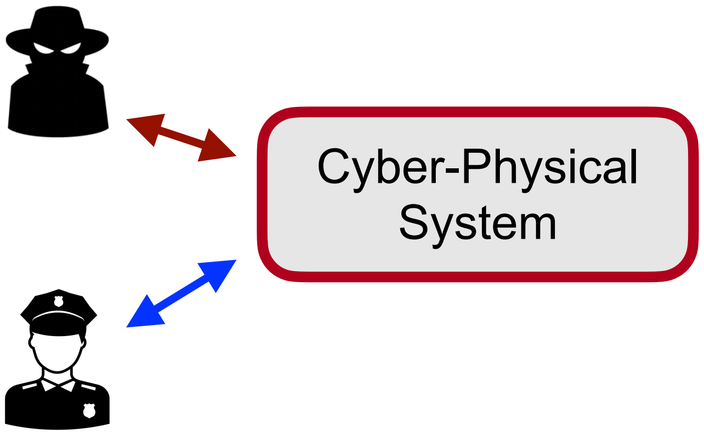
  - Modify the system dynamics



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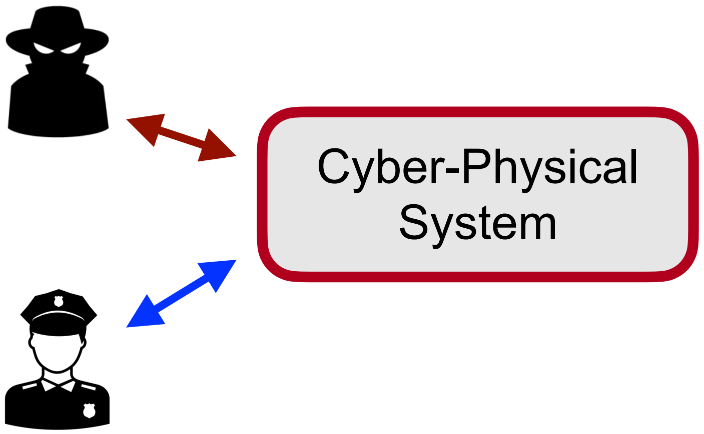
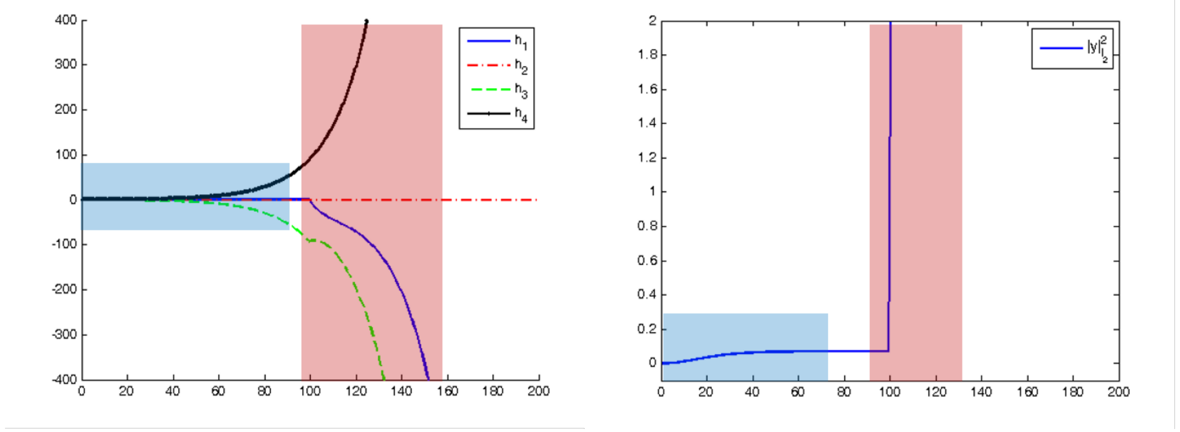


Example: connect Tank 1 to Tank 3



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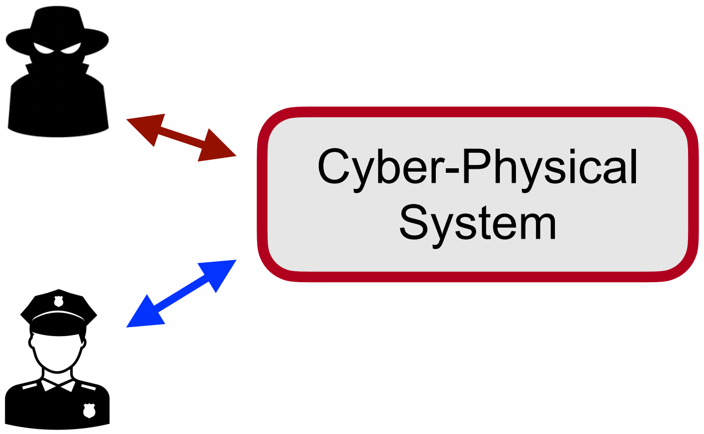

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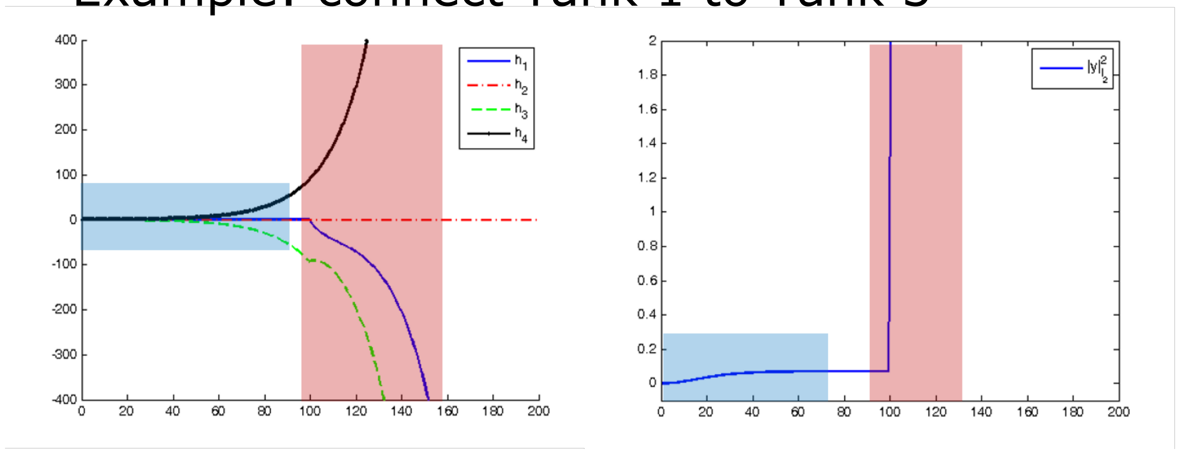

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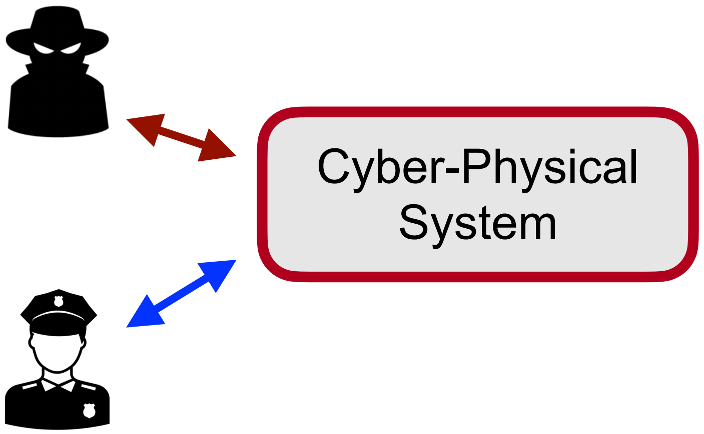

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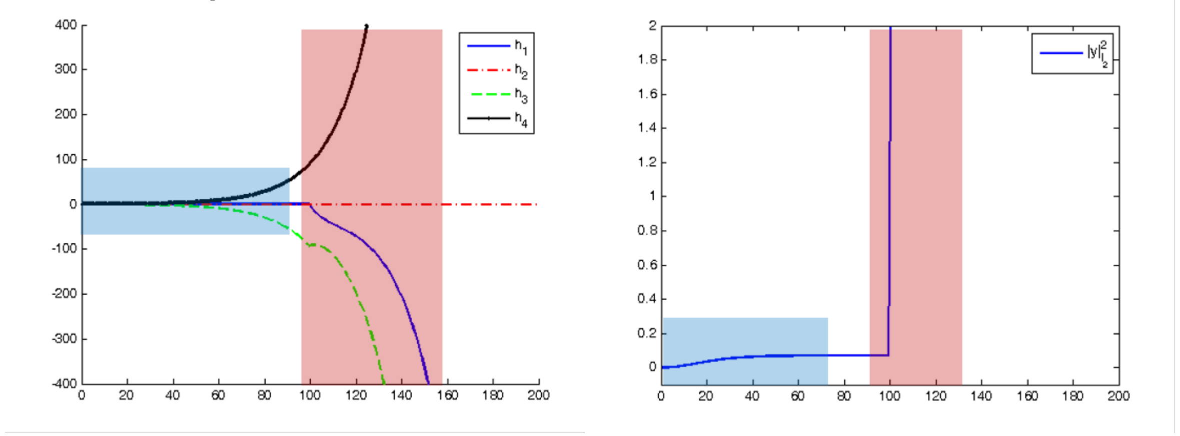

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The setting is uninformative of how secure the ‘new’ system is against ‘new’ attack policies!

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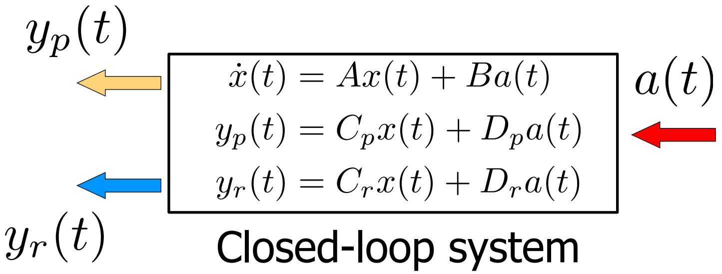
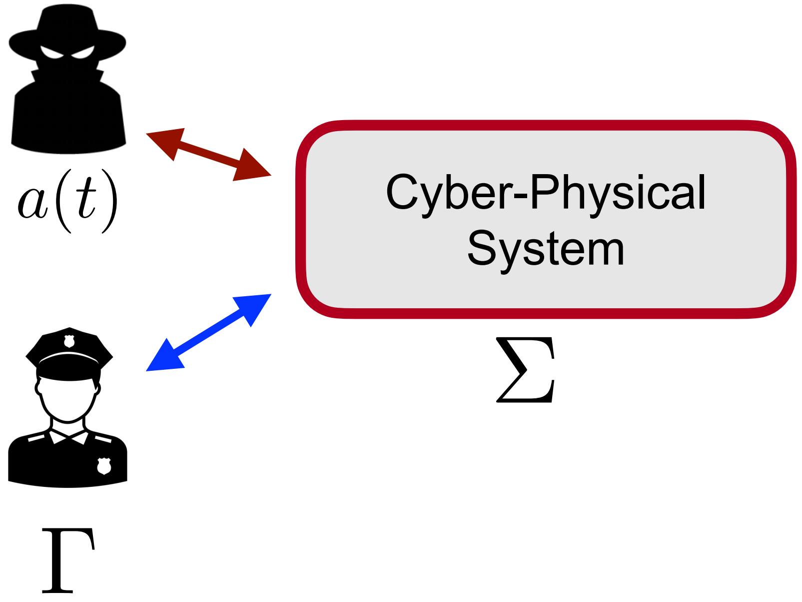
Security Metrics and Game-Theoretic Design





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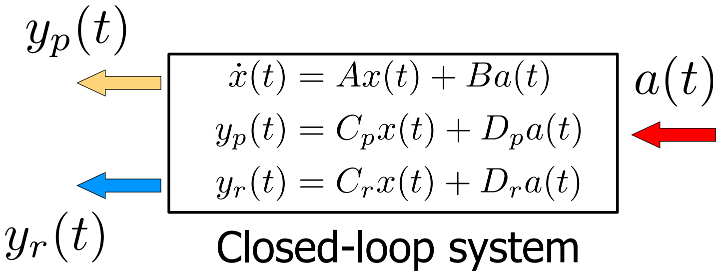
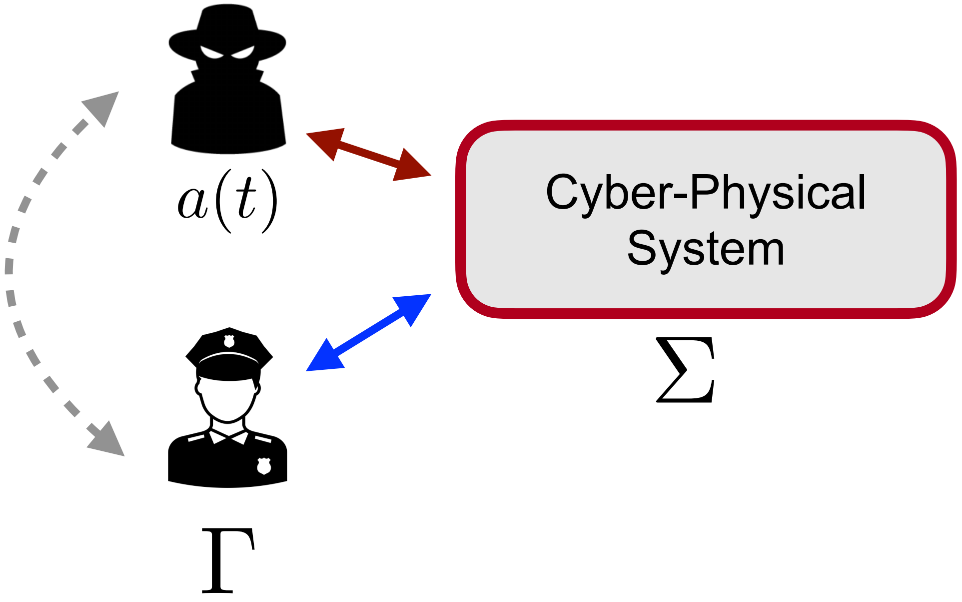


$$\min_{\Gamma} \max_{a} J(\Gamma, a; \Sigma)$$



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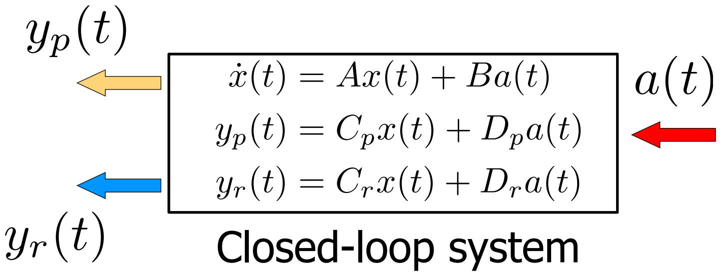

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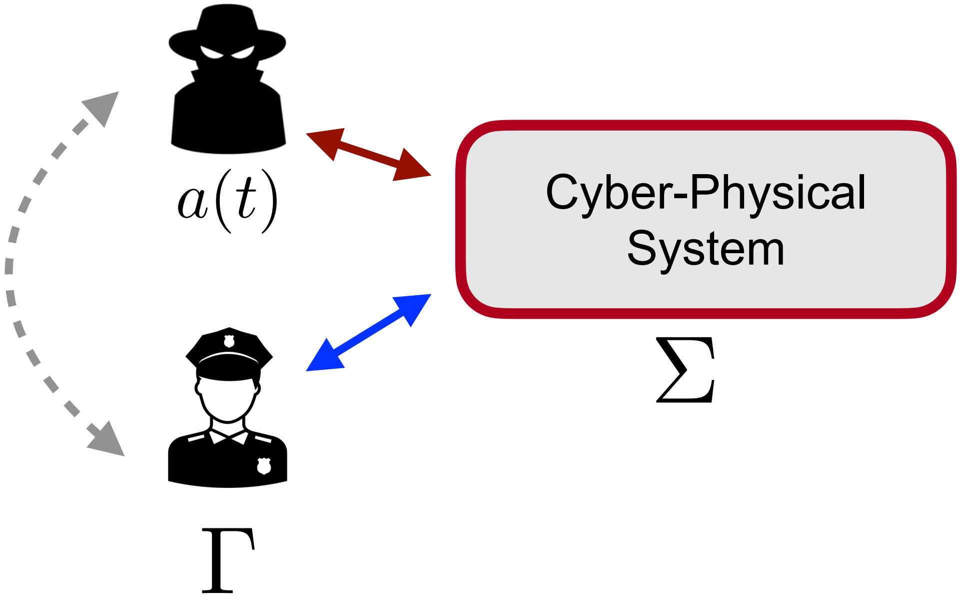
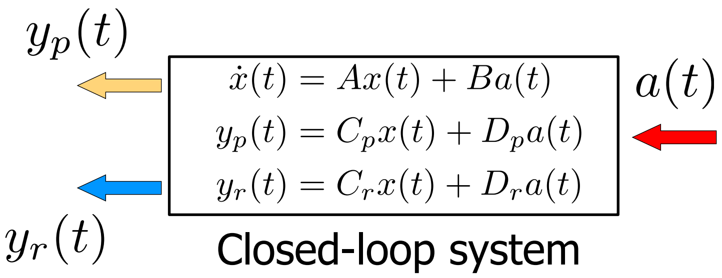


The security metric $J(\Gamma, a; \Sigma)$ captures the interactions between attacker and defender through the system. This enables us to construct a richer set of games between these players.

Security Metric for Control Systems



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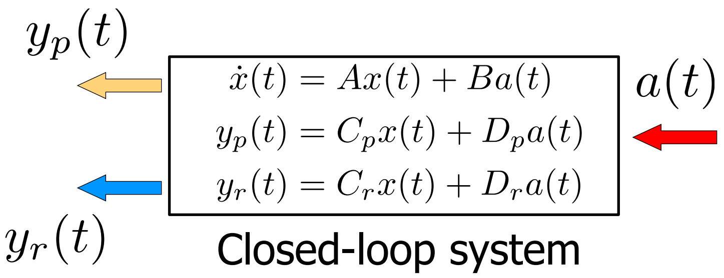
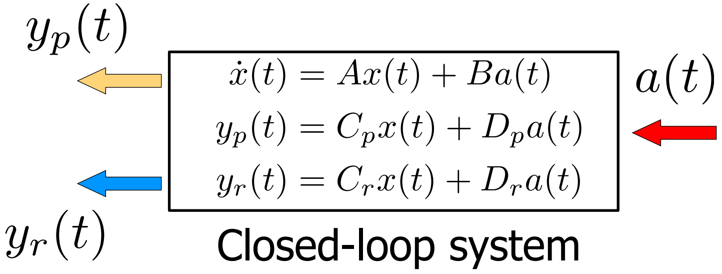


Attack policy: Maximise the impact on performance without raising alarms

Output-to-output gain: Maximize $\|y_p\|$, while keeping $\|y_r\|$ small

Security Metric for Control Systems



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$$\begin{aligned} \gamma^\star \triangleq \sup_{a \in \mathcal{L}_{2e}} \quad & \|y_p\|^2_{\mathcal{L}_2} \\ \text{s.t.} \quad & \|y_r\|^2_{\mathcal{L}_2} \leq 1 \\ & x(0) = 0 \end{aligned}$$

[Teixeira et al., CDC 15], [Teixeira, Springer 21]

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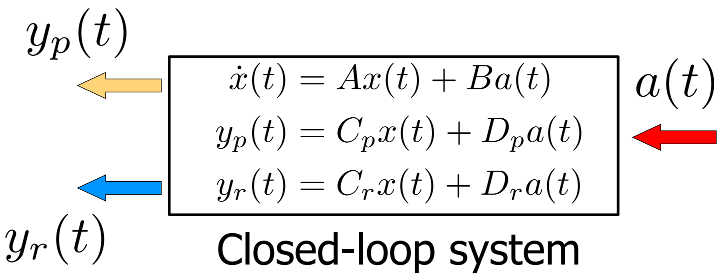

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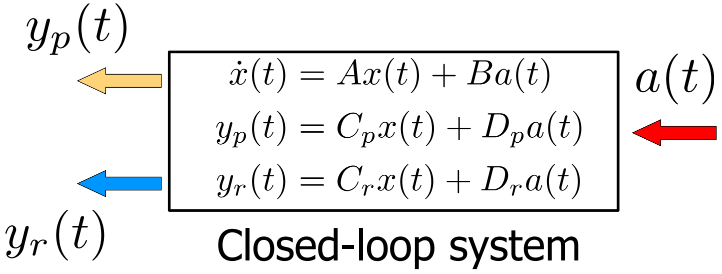
- Input is not directly constrained (may be exponentially increasing)

$\mathcal{L}_{2e}$ = "signals with finite energy over finite time intervals"

[Teixeira et al., CDC 15], [Teixeira, Springer 21]



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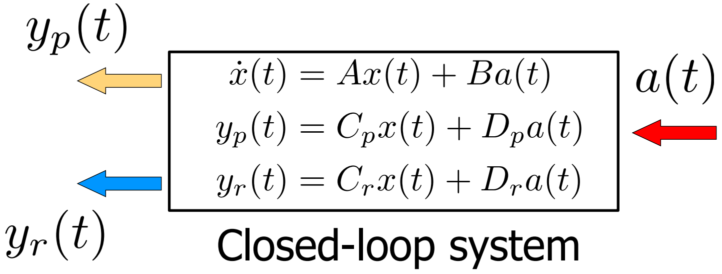
- 'Unstable zero dynamics' is an optimal policy



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- An equivalent formulation (dual problem):

$$\gamma^\star = \min_{\beta \geq 0} \quad \beta$$
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[Teixeira et al., CDC 15], [Teixeira, Springer 21]

Security Metric for Control Systems

$y_p(t)$ ← Closed-loop system ← $a(t)$

$y_r(t)$ ←

$$\dot{x}(t) = Ax(t) + Ba(t)$$
$$y_p(t) = C_p x(t) + D_p a(t)$$
$$y_r(t) = C_r x(t) + D_r a(t)$$

Closed-loop system

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- Finite $\gamma^\star$ implies a bound on the performance degradation by stealthy attacks

$$\|y_r\|_{\mathcal{L}_2}^2 \leq \theta \rightarrow \gamma^\star \theta \geq \|y_p\|_{\mathcal{L}_2}^2$$

[Teixeira et al., CDC 15], [Teixeira, Springer 21]

Security Analysis through Linear Matrix Inequalities

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(infinite-dimensional constraint)

The constraint can be re-cast as a Linear Matrix Inequality (LMI)

- Key technique: Dissipative Systems Theory (details in backup slides)

Can be efficiently solved by SDP solvers (e.g., through CVX)

$$\gamma^\star = \min_{\beta \geq 0,\, P \succeq 0} \quad \beta$$

$$\text{s.t.} \quad \begin{bmatrix} A^\top P + PA & PB \\ B^\top P & 0 \end{bmatrix} - \beta \begin{bmatrix} C_r^\top C_r & C_r^\top D_r \\ D_r^\top C_r & D_r^\top D_r \end{bmatrix} + \begin{bmatrix} C_p^\top C_p & C_p^\top D_p \\ D_p^\top C_p & D_p^\top D_p \end{bmatrix} \preceq 0$$

[Teixeira et al., CDC 15], [Teixeira, Springer 21]

Game-Theoretic Design through Bilinear Matrix Inequalities

Design problem for a Controller (L) and a Detector (K):

- K and L change the matrices of the closed-loop system

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\begin{aligned}
\min_{K,L} \sup_{a \in \mathcal{L}_{2e}} \quad & \|y_p\|^2_{\mathcal{L}_2} \\
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Designing a Controller (L) & Detector (K) is possible under specific forms

… but leads to Bilinear Matrix Inequalities [Teixeira, Springer 2021] [Anand and Teixeira, IFAC WC 2020]

$$\begin{aligned} \min_{P \succeq 0, \beta > 0, K, L} \quad & \beta \\ \text{s.t.} \quad & \begin{bmatrix} A(K,L)^\top P + PA(K,L) & PB(K,L) & C_p(K,L)^\top \\ B(K,L)^\top P & 0 & D_p(K,L)^\top \\ C_p(K,L) & D_p(K,L) & -\beta I \end{bmatrix} - \beta \begin{bmatrix} C_r{}^\top \\ D_r^\top \\ 0 \end{bmatrix} \begin{bmatrix} C_r & D_r & 0 \end{bmatrix} \preceq 0, \end{aligned}$$

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\end{aligned}
$$

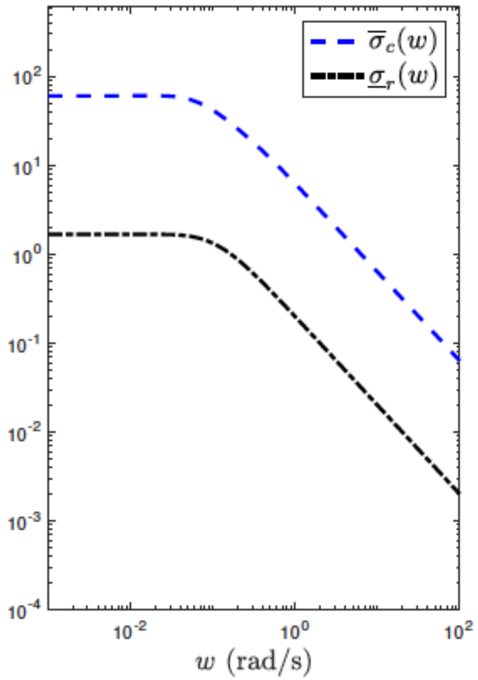
Next we use an heuristic: alternating minimisation

Example 1: Continuous-time

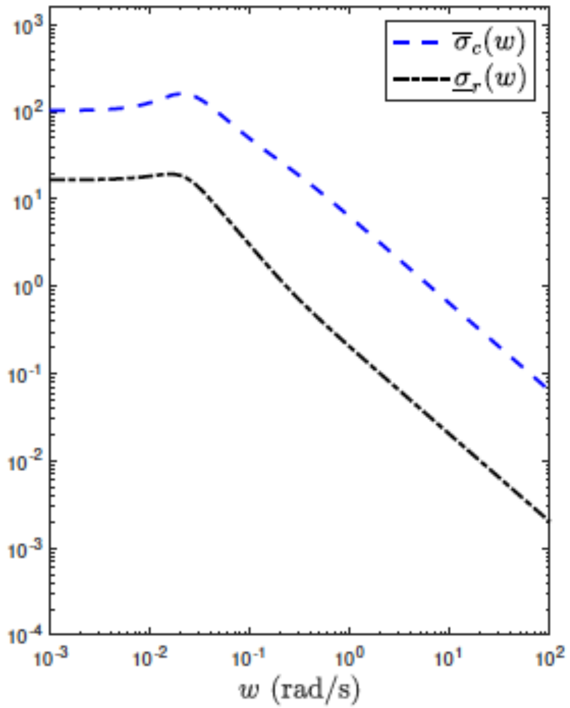
[Teixeira, Springer 2021]

- Classical vs Re-designed controller and detector

Nominal Design



After re-design



Example 2: Discrete-time

[Anand and Teixeira, IFAC WC 2020]

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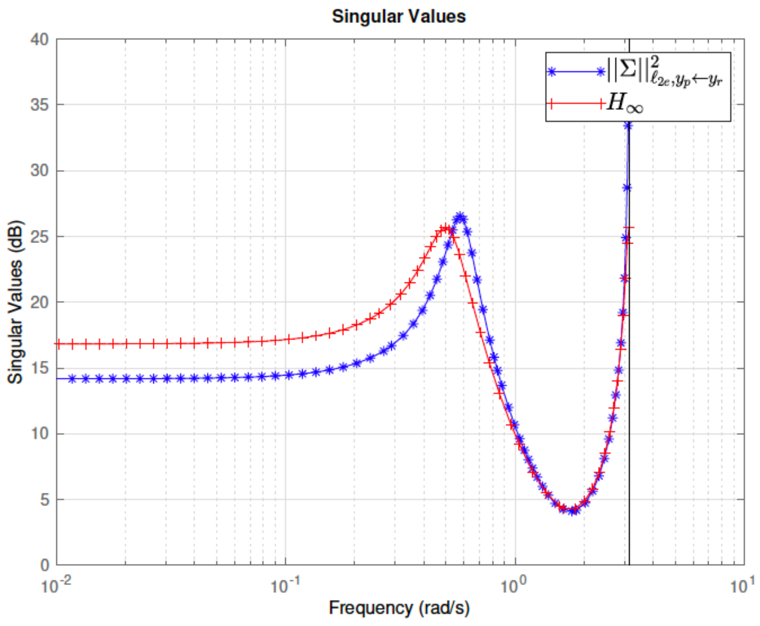

Fig. 1. Singular values - Performance output $(\bar{\sigma}(\Sigma_p))$

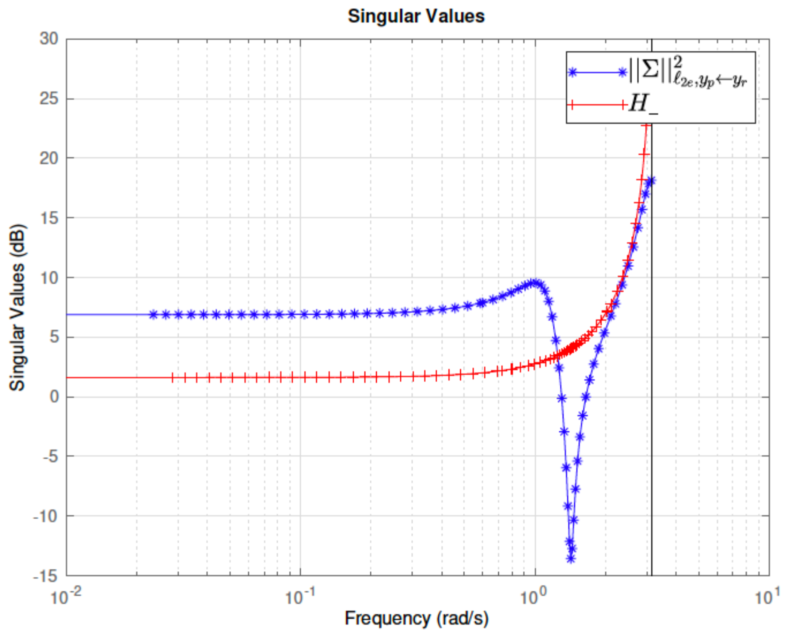

Fig. 2. Singular values - Detection output $(\underline{\sigma}(\Sigma_r))$

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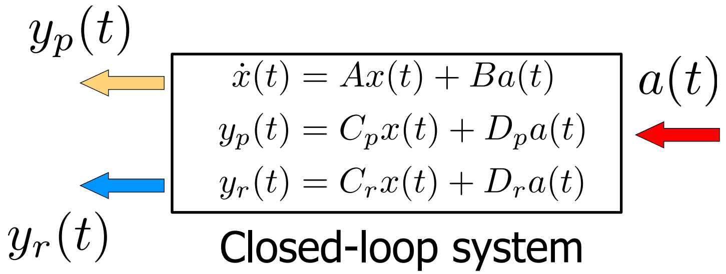

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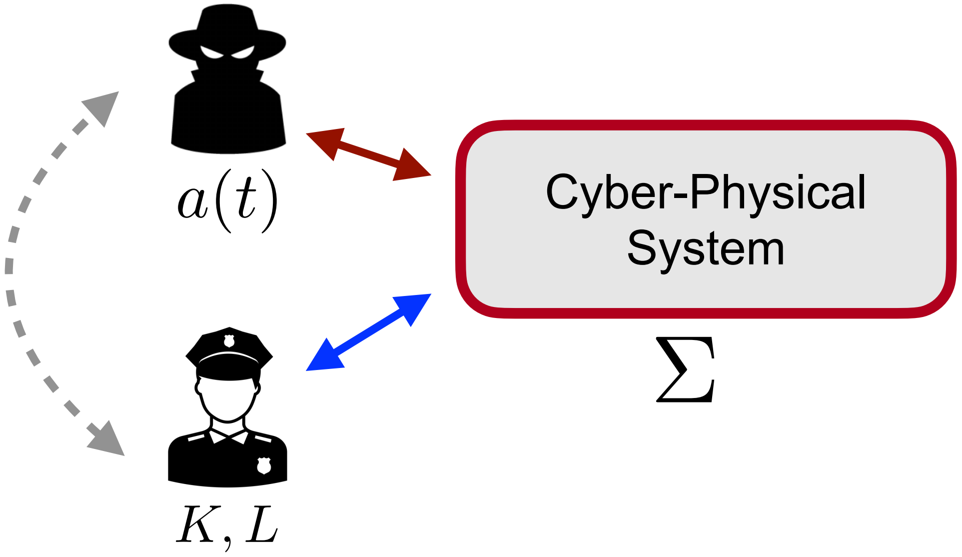

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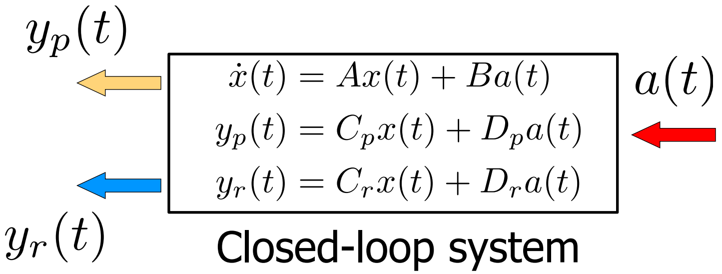

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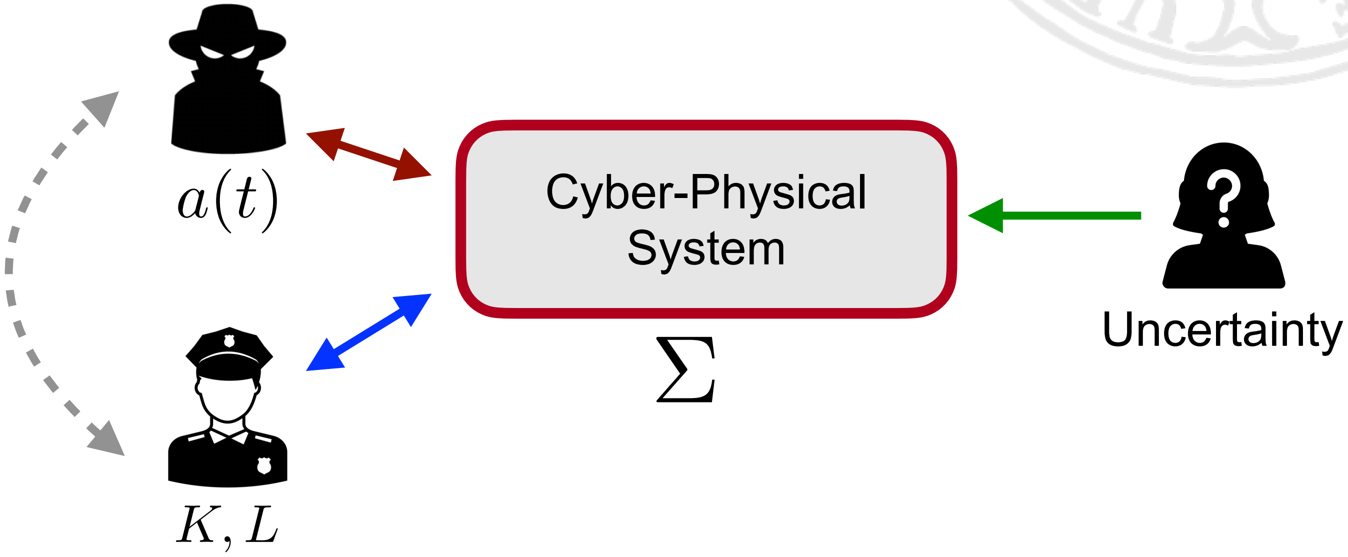
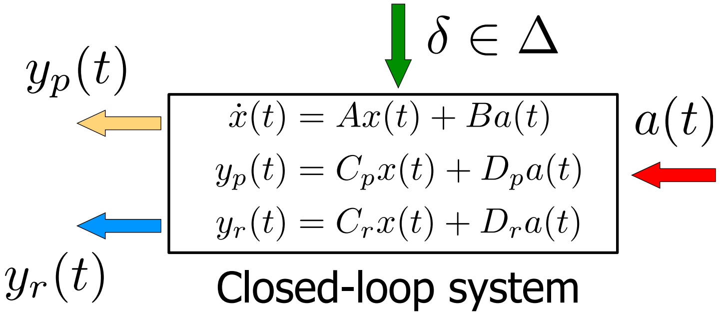
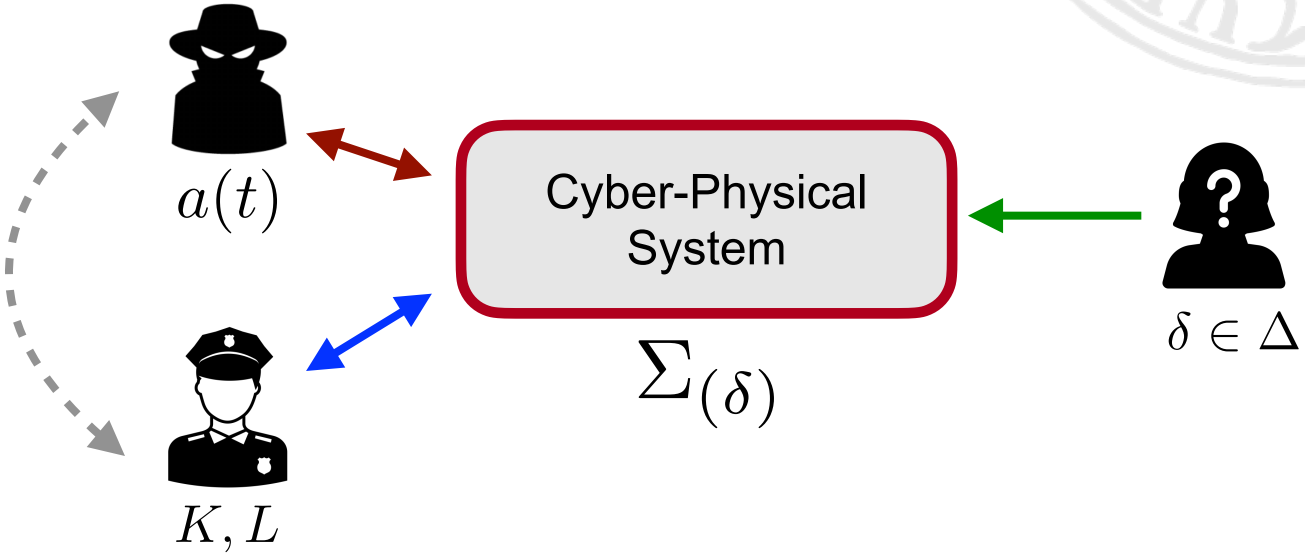


Outline

- Security Risk Management
- Scenario and Threat Models
- Security Metrics and Game-Theoretic Design
- **Security under Model Uncertainty**
- Probabilistic Risk Measures and Game-Theoretic Design
- Conclusions and Remarks

Security Metrics under Model Uncertainty





$$
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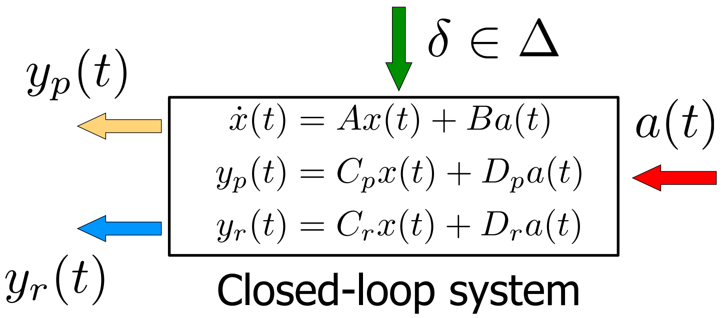
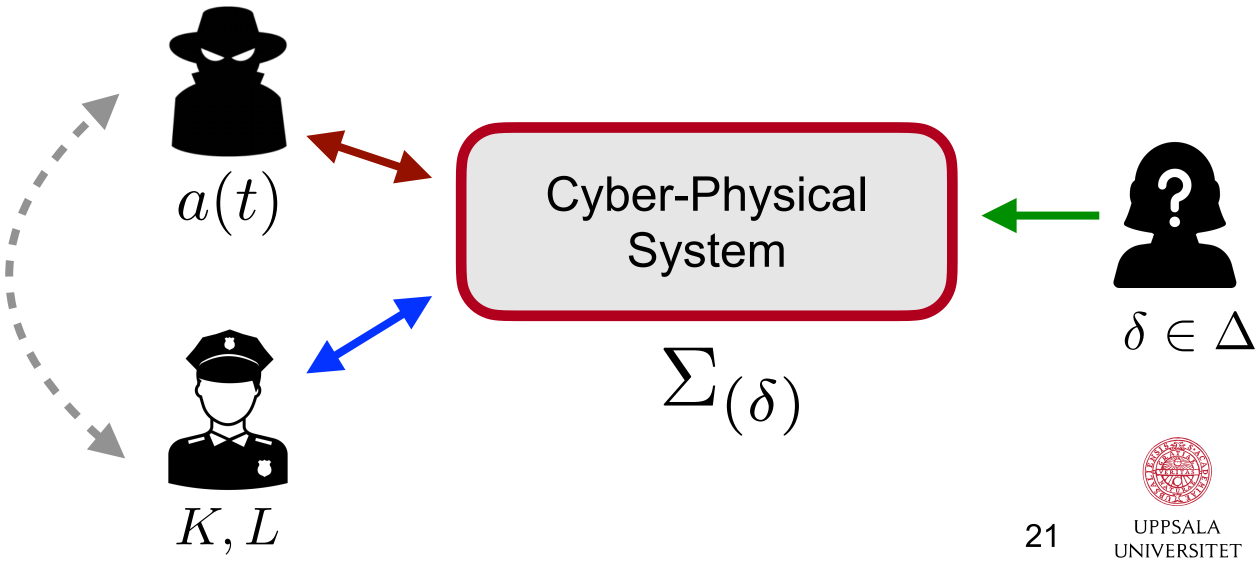
$$

How should uncertainty be embedded in the Defender and Adversary?

Worst-Case Model Uncertainty





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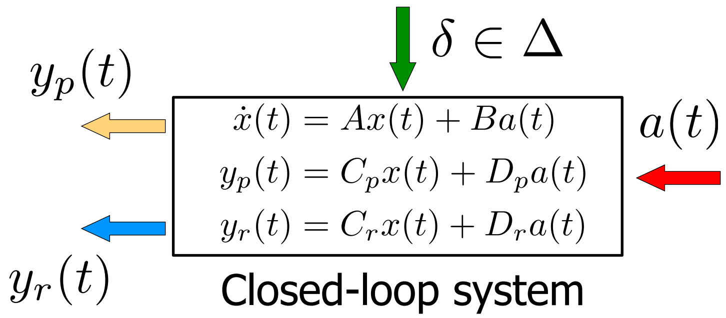

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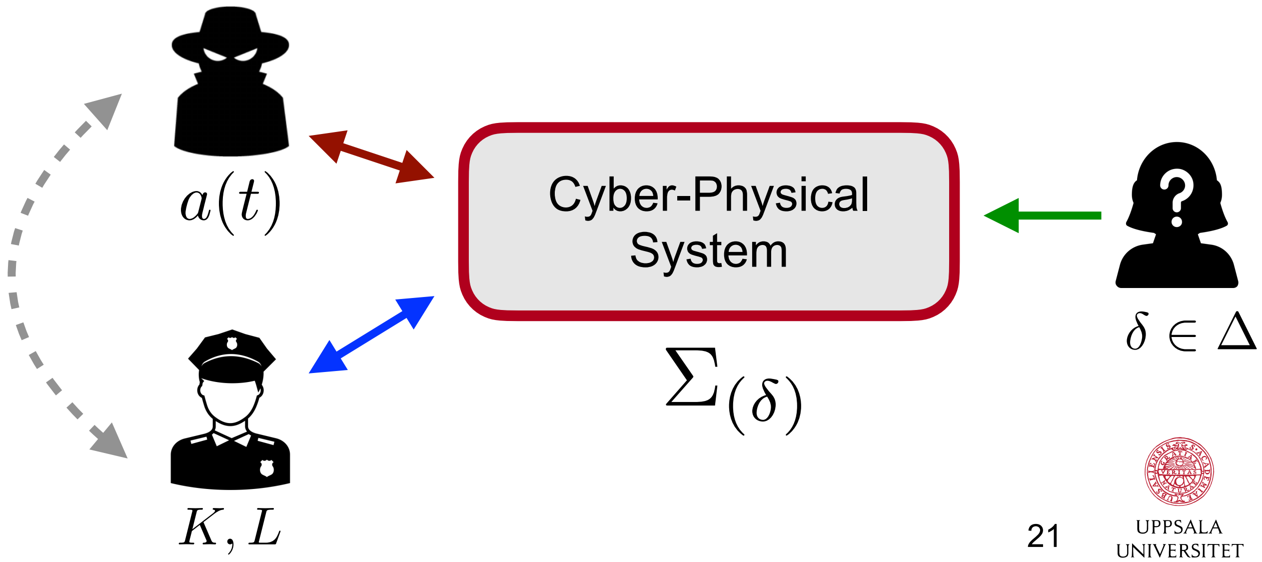

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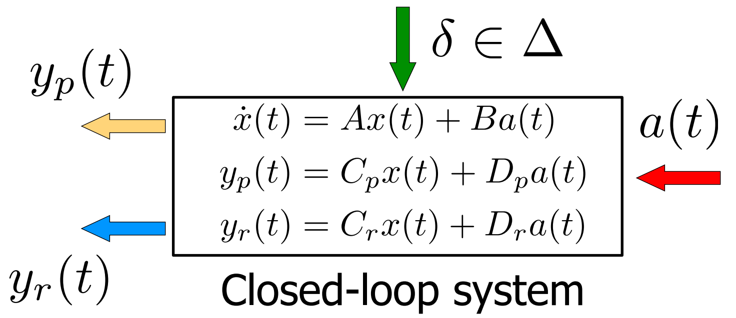

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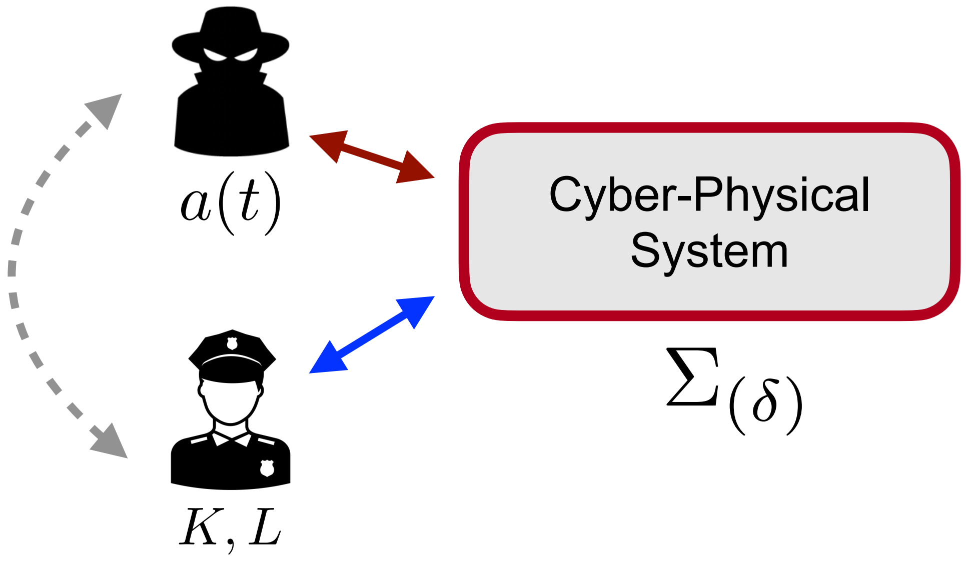

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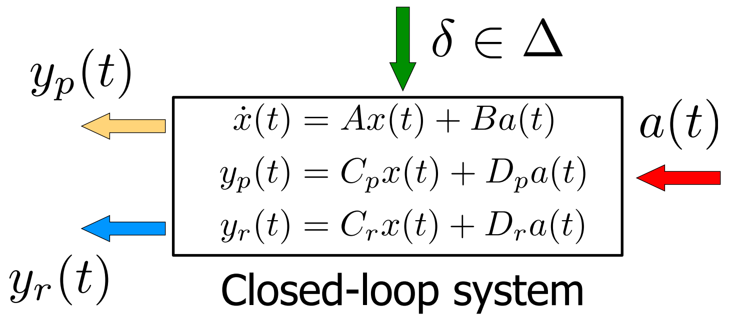

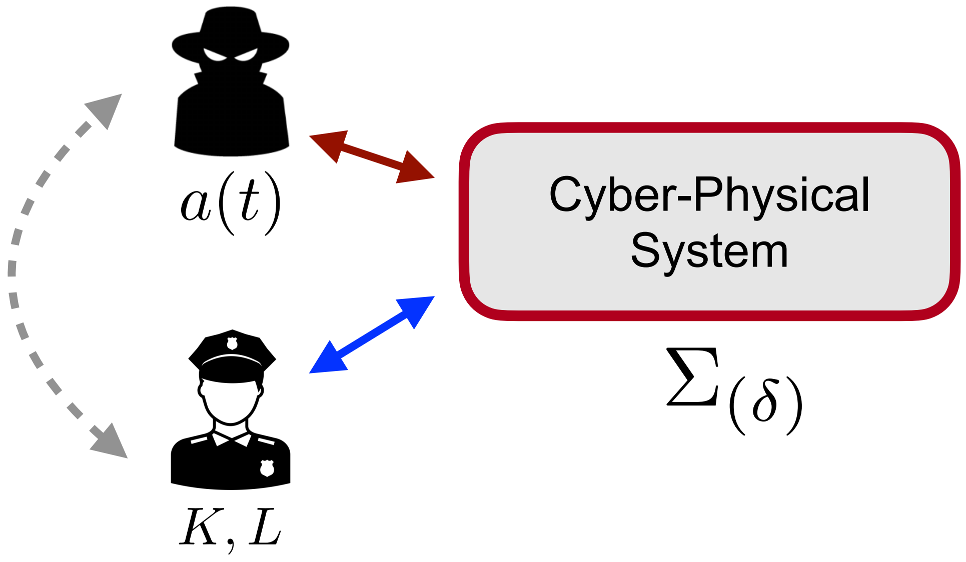

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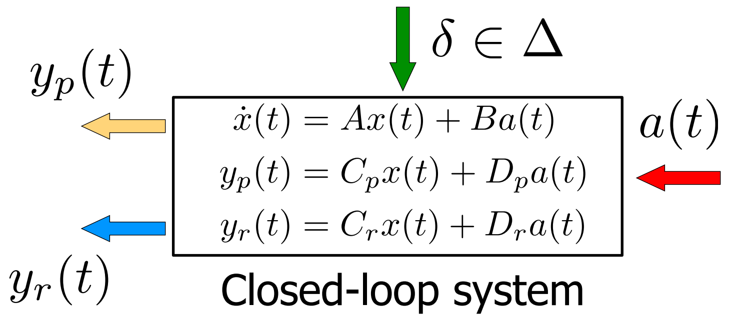

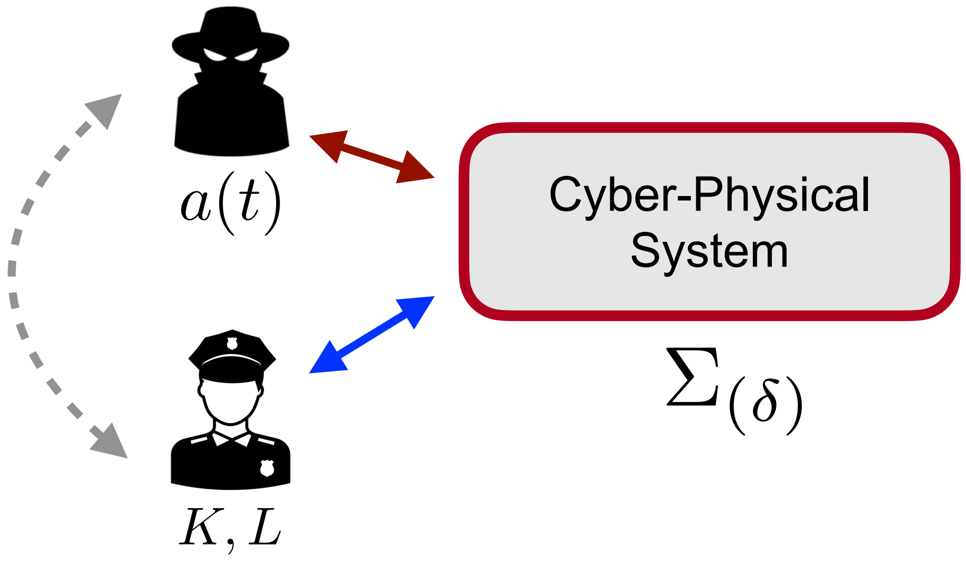

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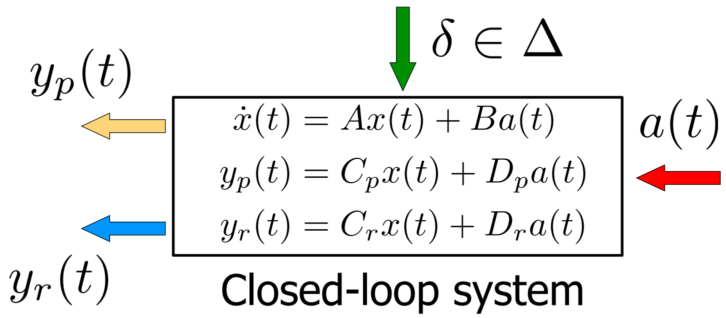

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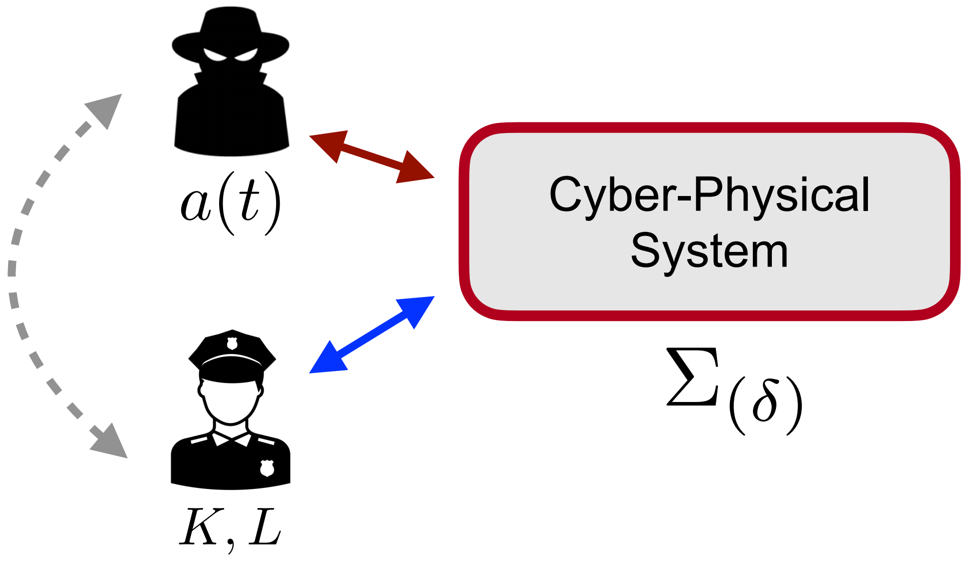

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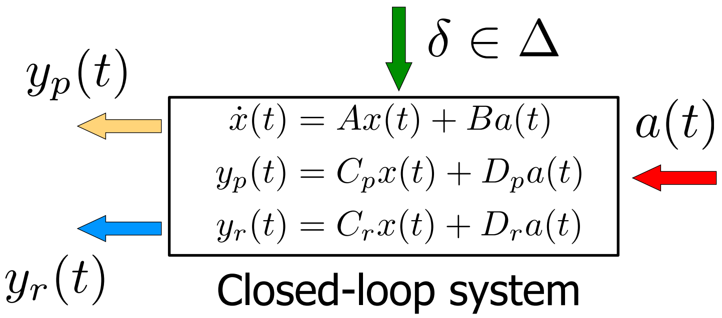

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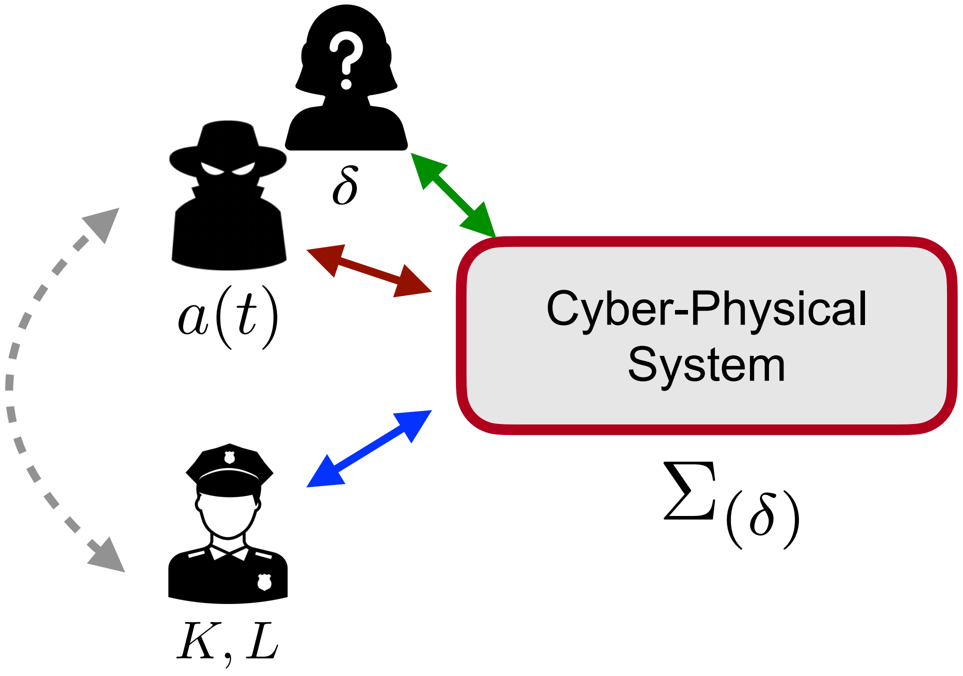
Adversary and Uncertainty are colluding!

Uncertainty “reacts” to defender’s actions



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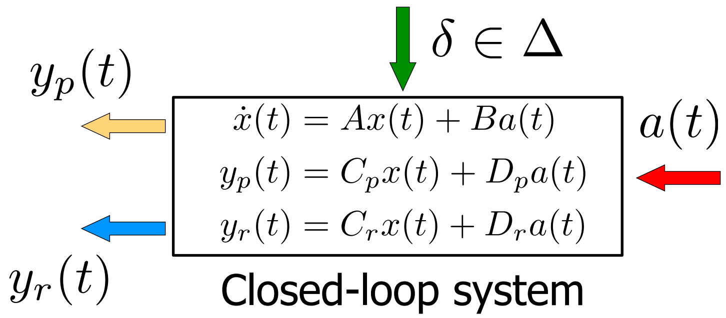

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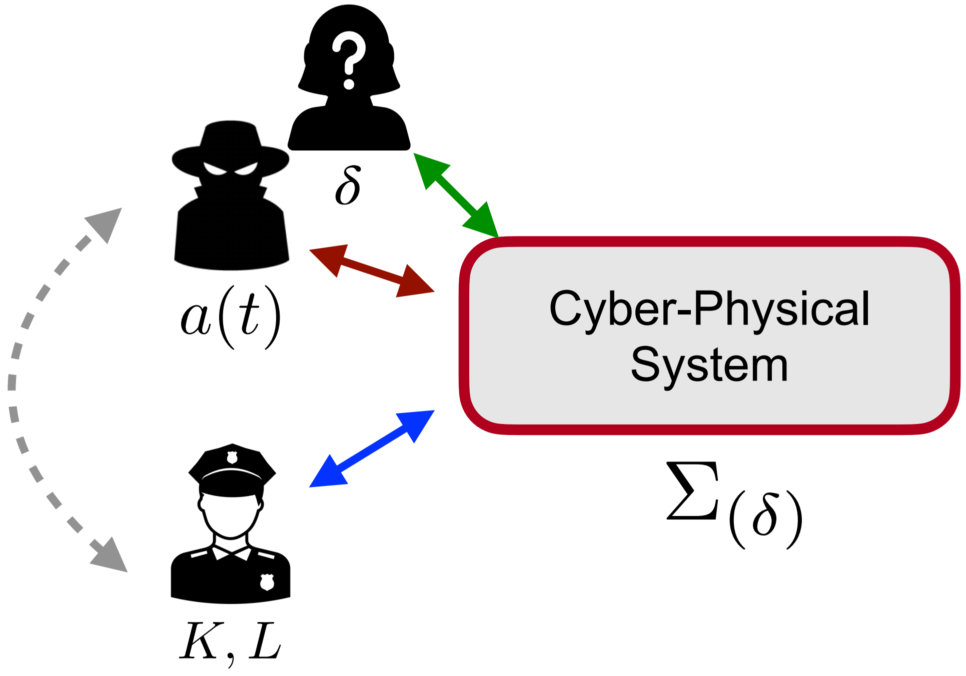
\begin{aligned}
\gamma^\star = \min_{\beta \geq 0} \quad & \beta \\
\text{s.t.} \quad & \beta \|y_r\|_{\mathcal{L}_2}^2 - \|y_p\|_{\mathcal{L}_2}^2 \geq 0, \ \forall\, a \in \mathcal{L}_{2e}, \ \forall \delta \in \Delta
\end{aligned}
$$

Adversary and Uncertainty are colluding!

Uncertainty “reacts” to defender’s actions

As in robust control, worst-case disturbance can be conservative!

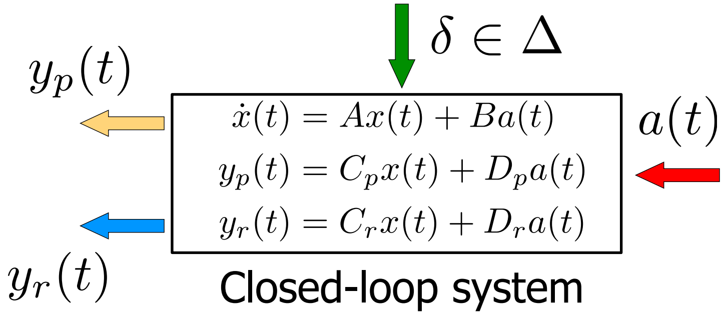
G. C. Calafiore and M. C. Campi, "The scenario approach to robust control design," *IEEE TAC*, 2006

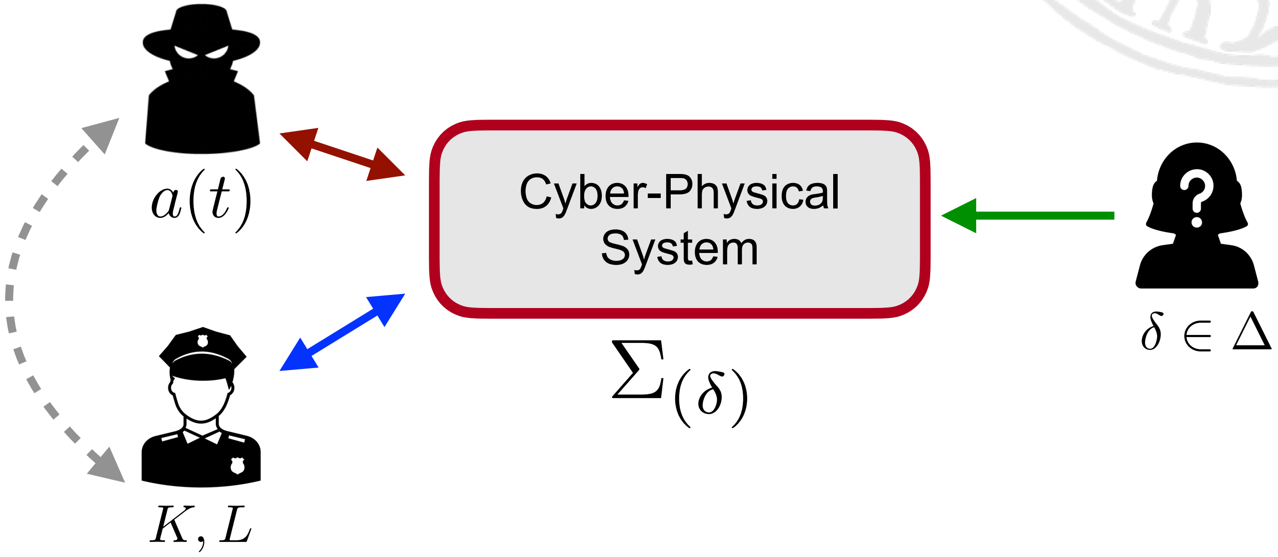


Outline

- Security Risk Management
- Scenario and Threat Models
- Security Metrics and Game-Theoretic Design
- Security under Model Uncertainty
- **Probabilistic Risk Measures and Game-Theoretic Design**
- Conclusions and Remarks

Security Metrics under Probabilistic Model Uncertainty





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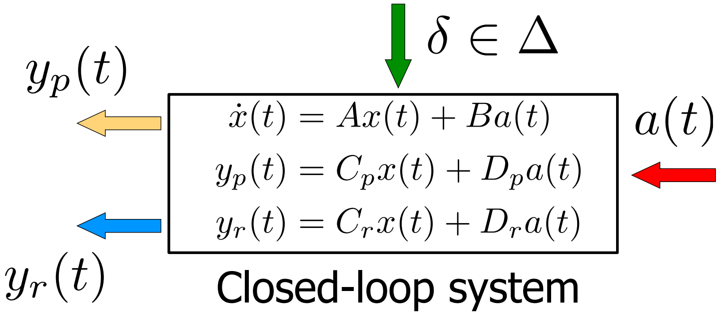

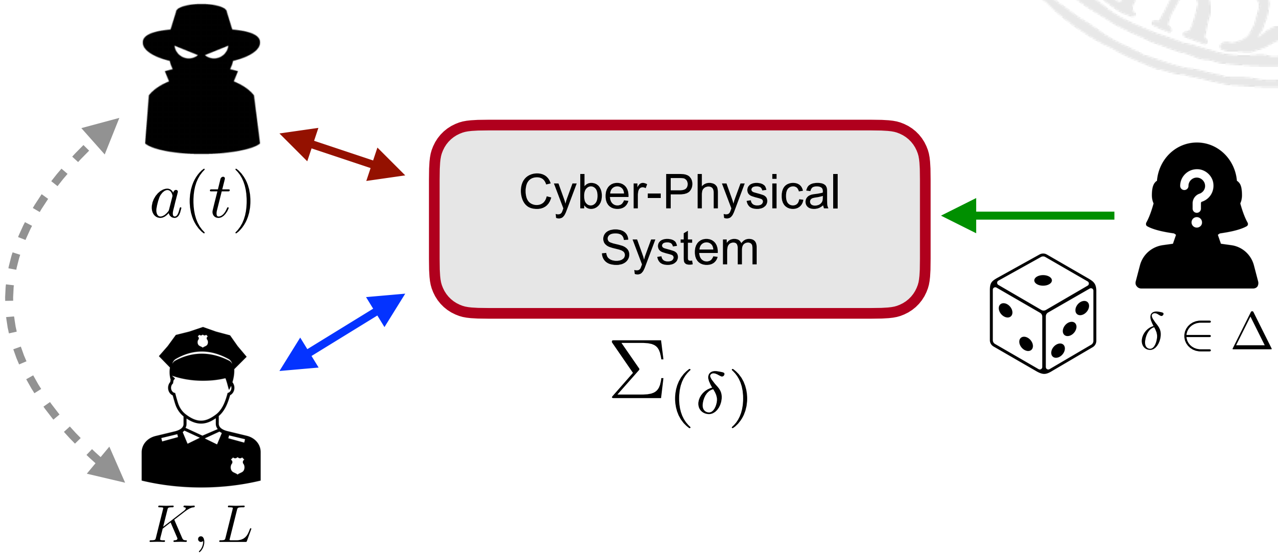

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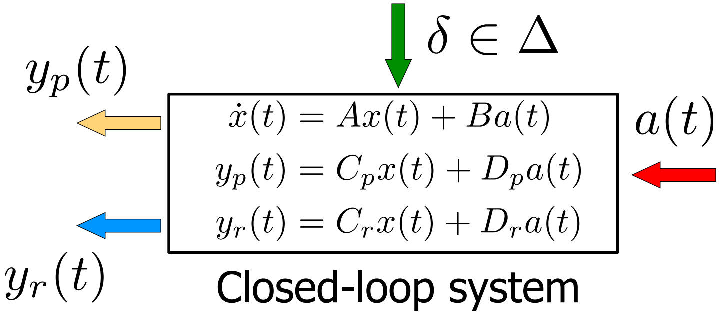
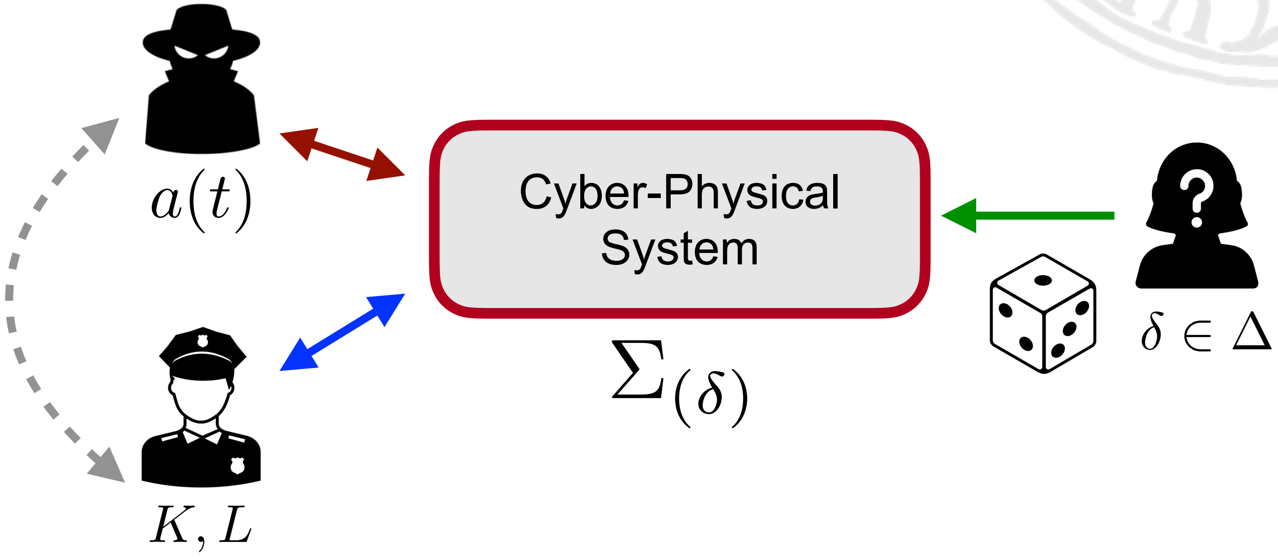
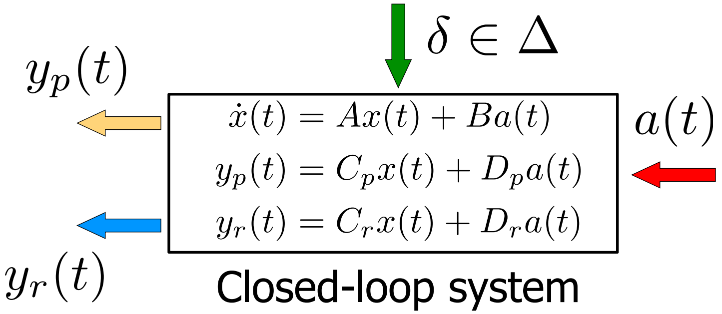
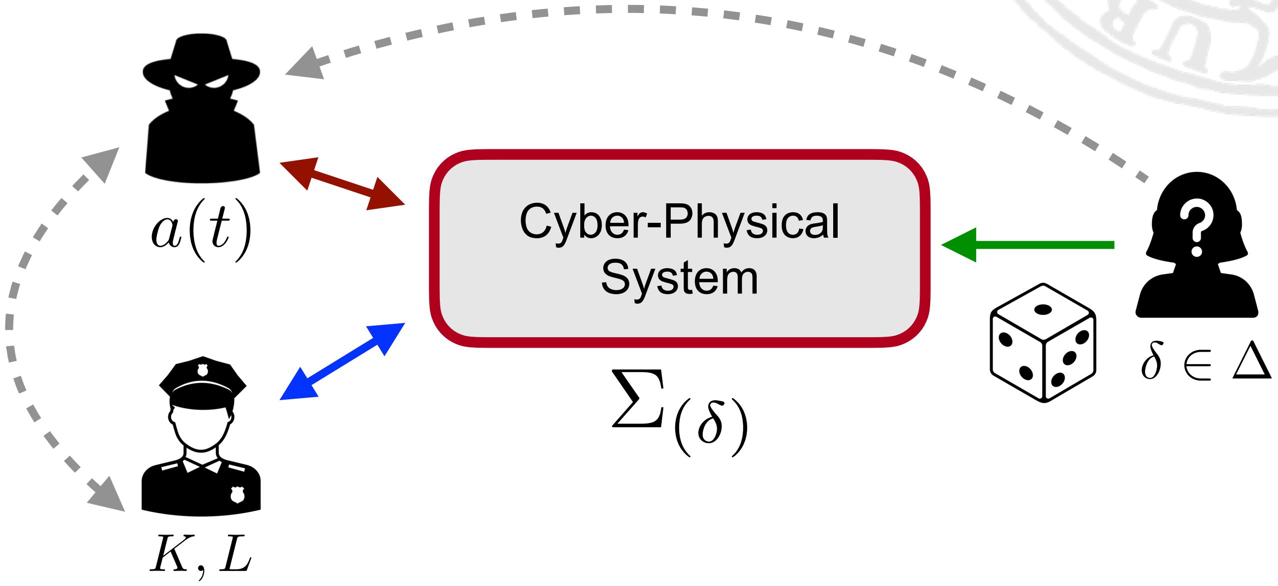




What is the information structure between the Uncertainty and the Adversary?

Security Metrics under Probabilistic Model Uncertainty





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(i) Anand, Teixeira. "Risk-based Security Measure Allocation Against Actuator Attacks". IEEE Open Journal of Control Systems, 2023.

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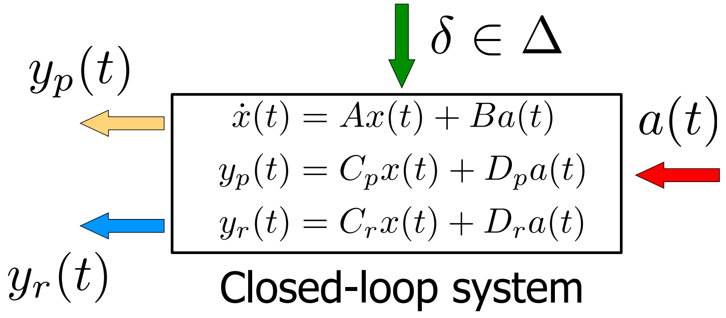
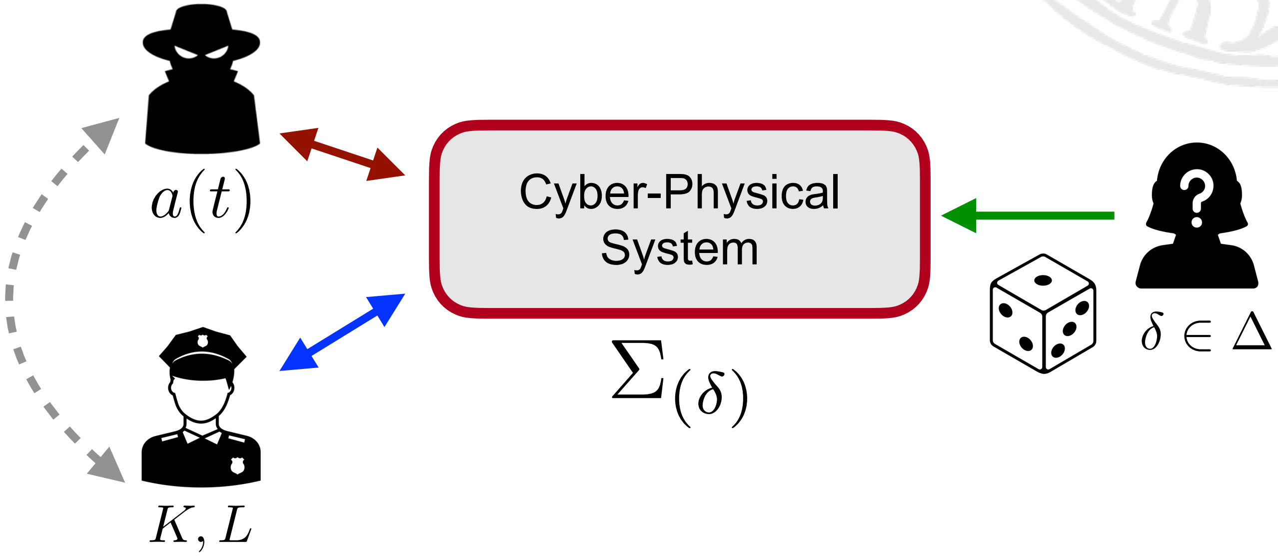




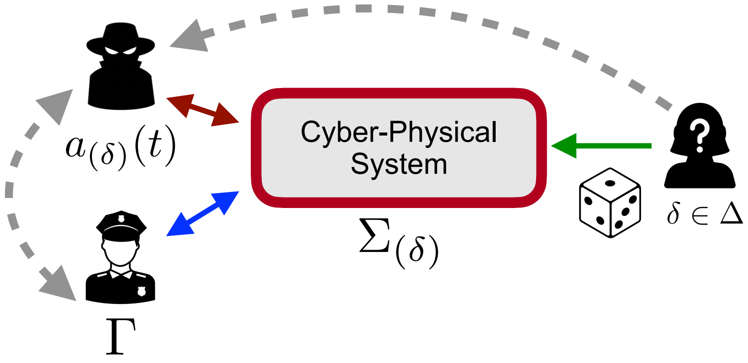
What is the <u>information structure</u> between the Uncertainty and the Adversary?

(i) **Omniscient Adversary:** knows the realization of the uncertainty, but they do not collude.

(ii) **Imperfect-information Adversary:** does not know the realization, needs to be robust to the uncertainty

(i) Anand, Teixeira. "Risk-based Security Measure Allocation Against Actuator Attacks". IEEE Open Journal of Control Systems, 2023.
(ii) Anand et al.. "Risk Assessment of Stealthy Attacks on Uncertain Control Systems". IEEE TAC, 2023

Probabilistic Risk Measures and Game-Theoretic Design



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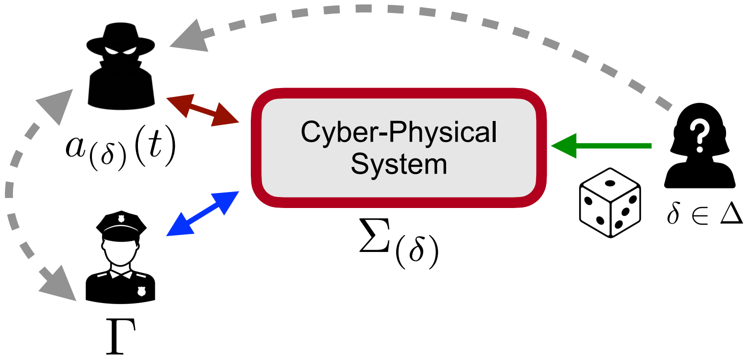
Probabilistic Risk Measures and Game-Theoretic Design



Impact of an **Omniscient Adversary:**

$$
\begin{aligned}
q(\Gamma, \delta) \triangleq \sup_{a_{(\delta)} \in \mathcal{L}_{2e}} \quad & \|y_{p,(\delta)}\|^2_{\mathcal{L}_2} \\
\text{s.t.} \quad & \|y_{r,(\delta)}\|^2_{\mathcal{L}_2} \leq 1 \\
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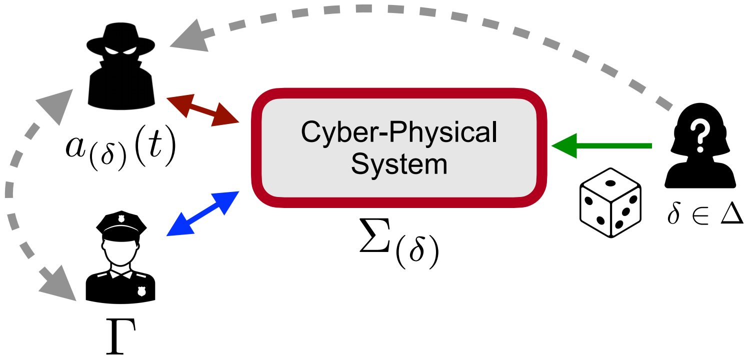

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The impact $q(\Gamma,\delta)$ is a random variable with a distribution induced by the uncertainty.

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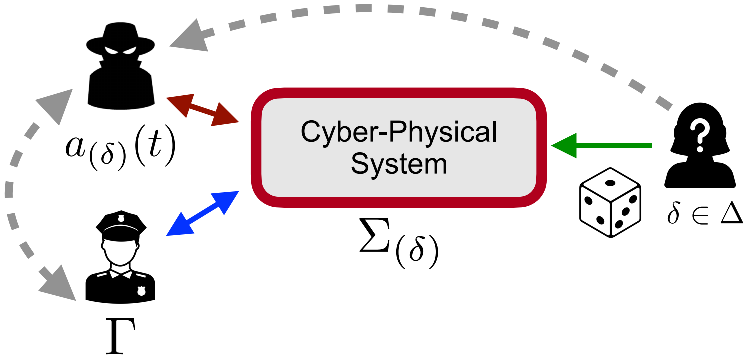
Probabilistic Risk Measures and Game-Theoretic Design



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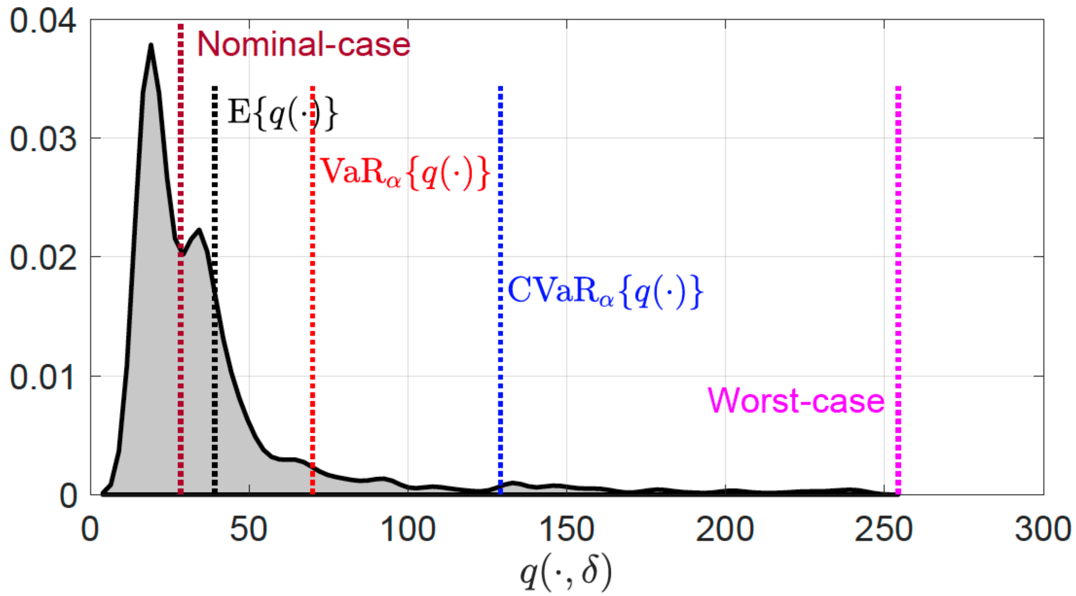

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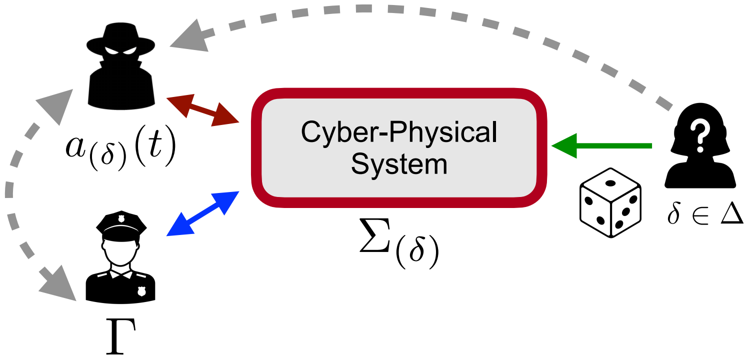
Probabilistic Risk Measures and Game-Theoretic Design



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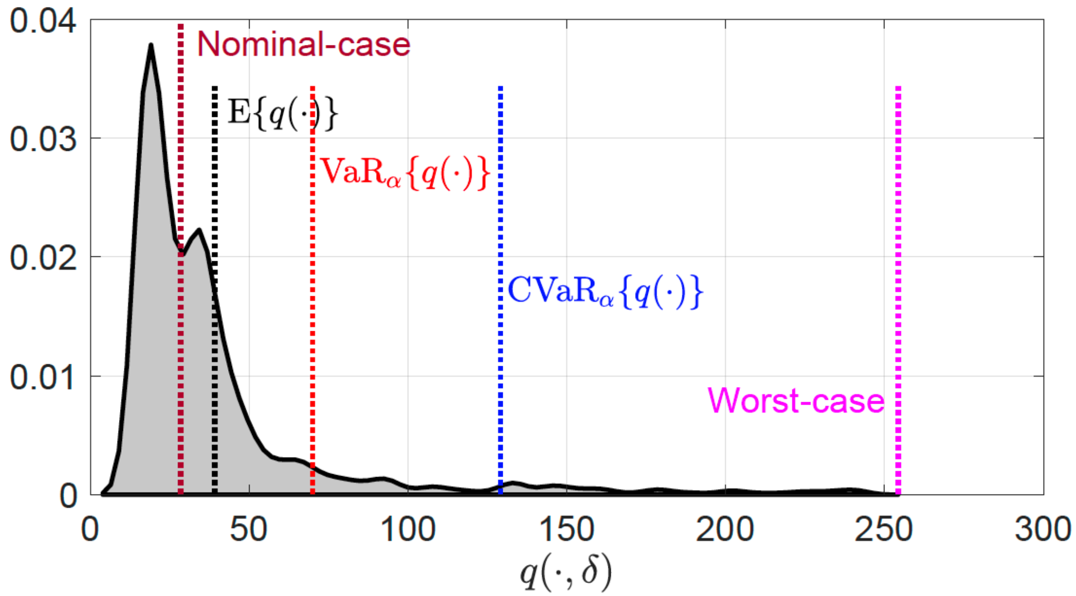

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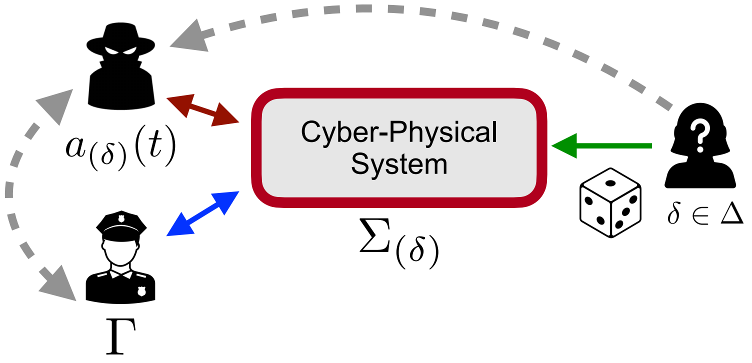

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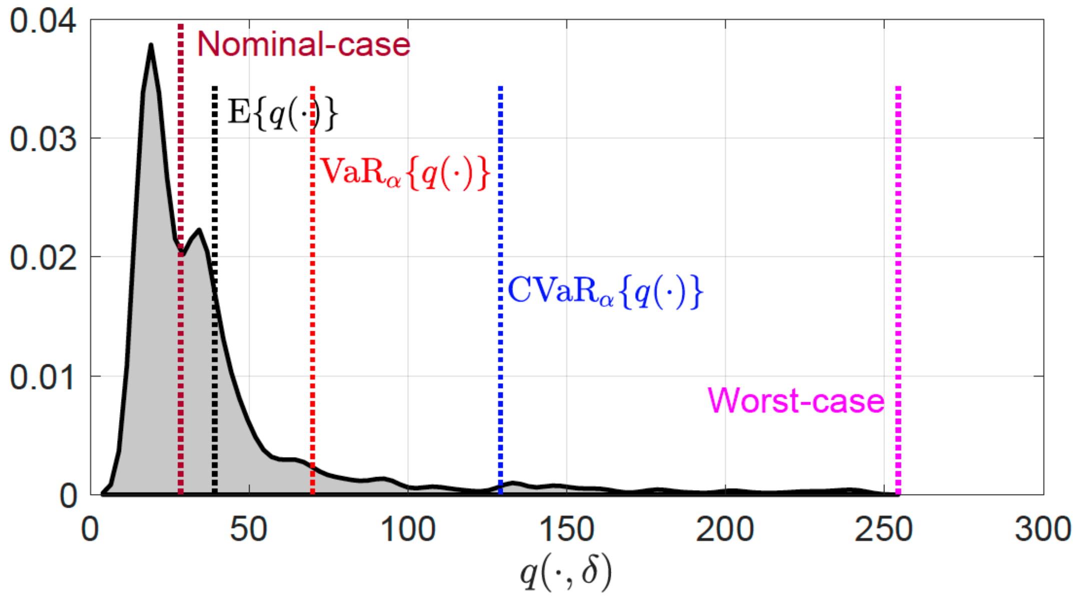

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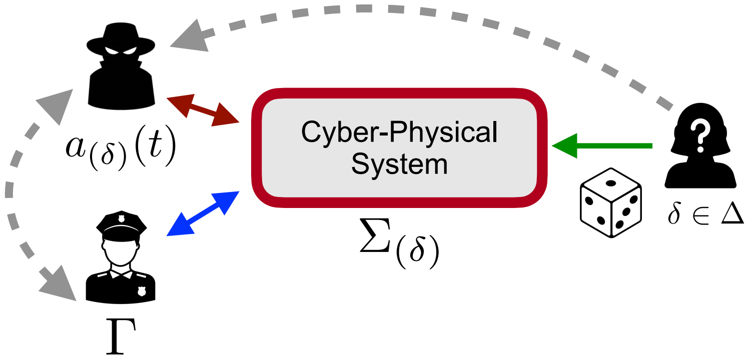
Example: allocation of protection on actuator channels



Γ - set of protected actuators

$$
\begin{aligned}
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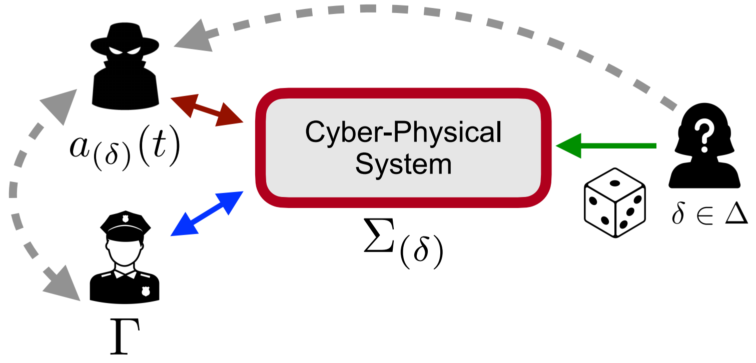

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The **risk measure** $R_{\Delta}\{\,\cdot\,\}$ is chosen as the CVaR, using sample-based approximations.

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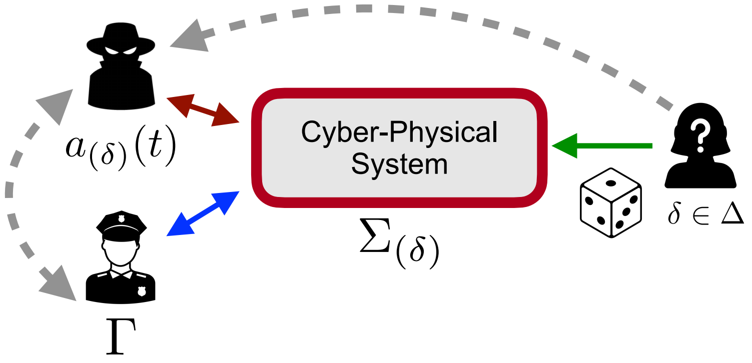

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Risk-optimal defense: $\min_{|\Gamma| < n_w} R_\Delta \{q(\Gamma, \delta)\}$

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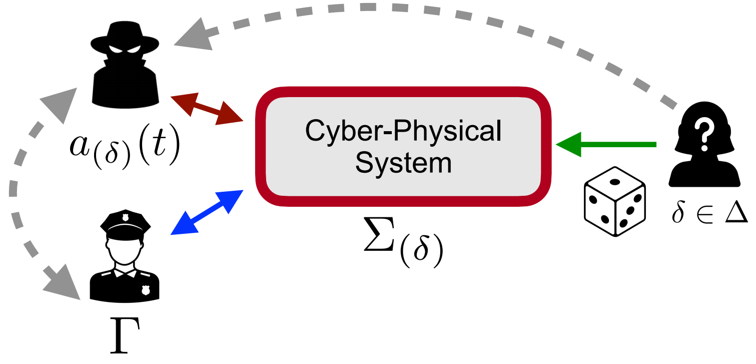

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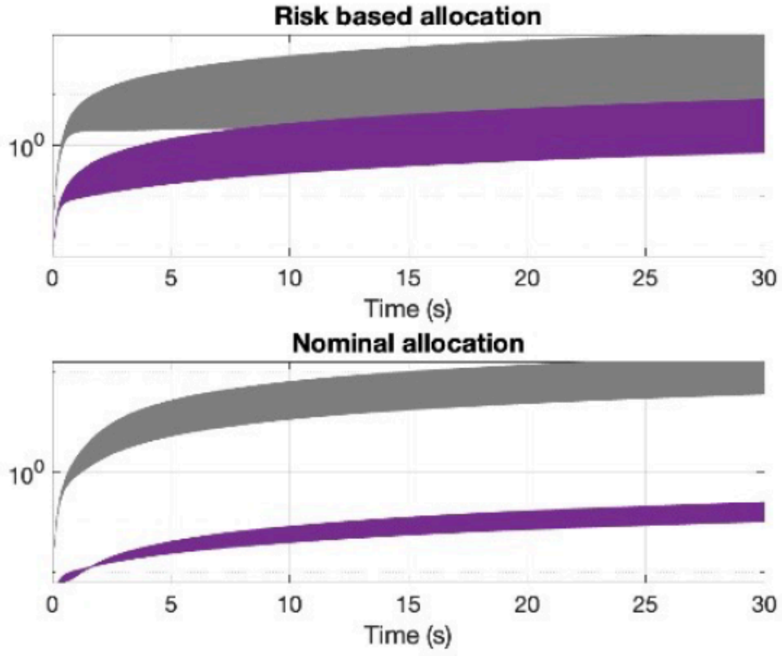

FIGURE 5. Performance energy (grey) and detection energy (violet) for $N = 500$ different realizations of uncertainty, under CVaR-based allocation strategy (top), and the nominal allocation strategy (bottom).

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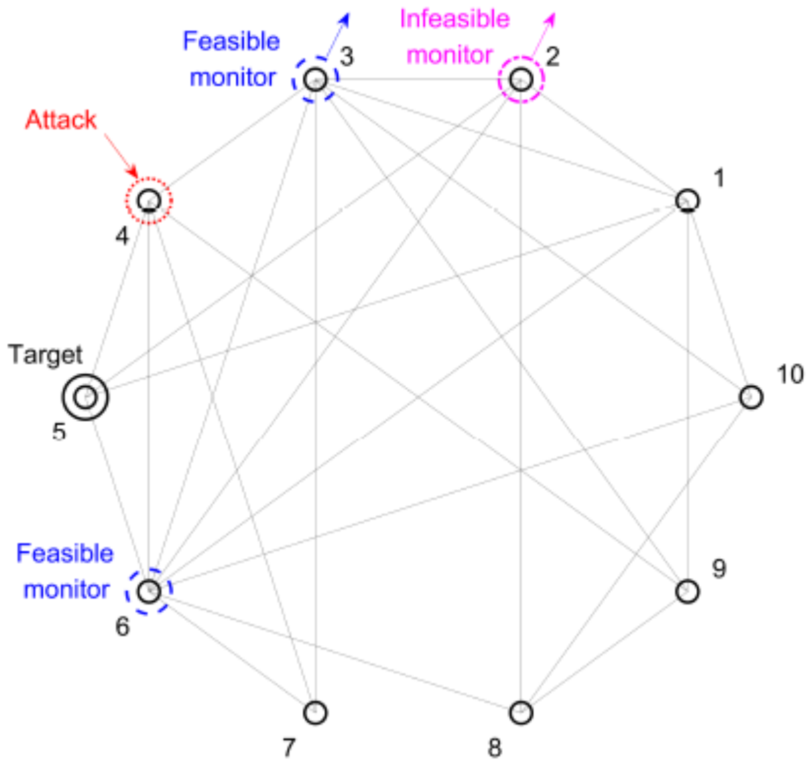
Anand, Teixeira. “Risk-based Security Measure Allocation Against Actuator Attacks”. IEEE Open Journal of Control Systems, 2023.

Other Security Games with Omniscient Adversaries

Security Allocation in Uncertain Large-Scale Systems

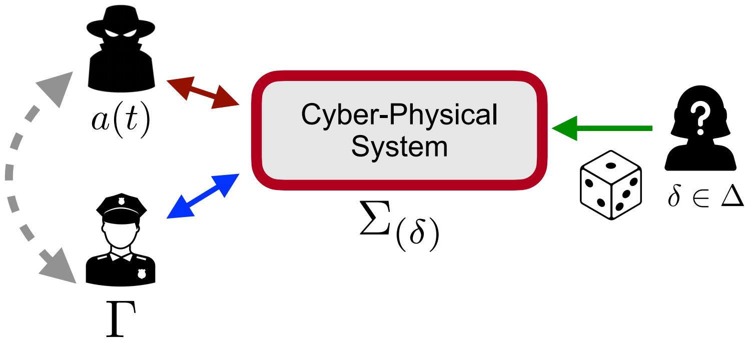
- Stackelberg games
  - Nguyen, et al. “Security Allocation in Networked Control Systems under Stealthy Attacks”. Submitted IEEE TCNS, 2023.
- Mixed Nash solutions and probabilistic uncertainty
  - Nguyen, et al. “A Zero-Sum Game Framework for Optimal Sensor Placement in Uncertain Networked Control Systems under Cyber-Attacks”. CDC 2022



Risk-averse controller design

- Anand et al. “Risk-averse controller design against data injection attacks on actuators for uncertain control systems”. ACC 2022

Robust attacks under probabilistic uncertainty



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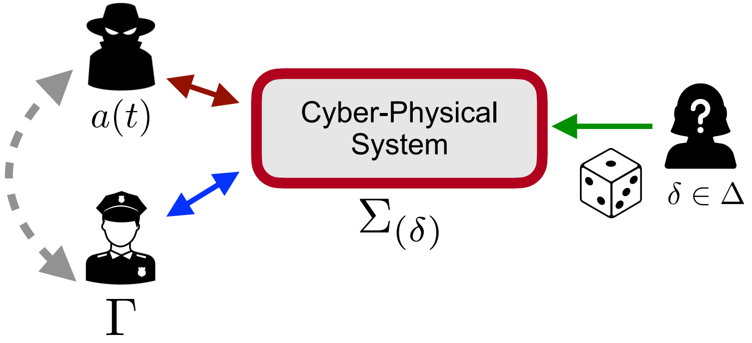

Risk of a **Imperfect-information Adversary:**

$$R_{\hat{\Delta}_N}(\Gamma) \triangleq \sup_{a \in \mathcal{L}_{2e}} \quad \frac{1}{N} \sum_{\delta \in \hat{\Delta}_N} \|y_{p,(\delta)}\|^2_{\mathcal{L}_2}$$

$$\text{s.t.} \quad \|y_{r,(\delta)}\|^2_{\mathcal{L}_2} \leq 1, \ \forall \delta \in \hat{\Delta}_N$$

Anand et al.. "Risk Assessment of Stealthy Attacks on Uncertain Control Systems". IEEE TAC, 2023

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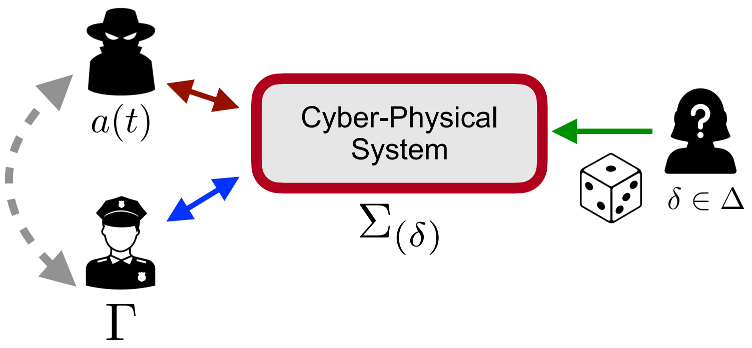

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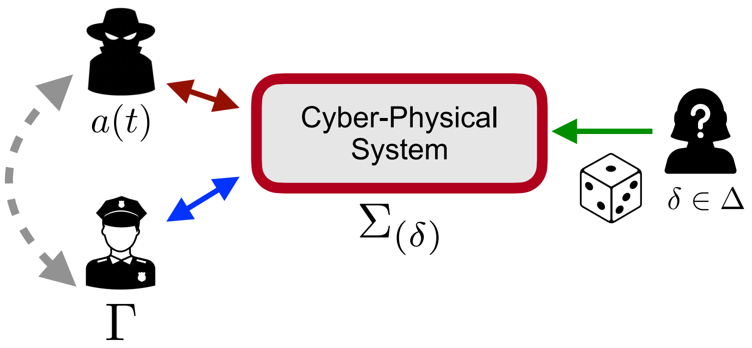

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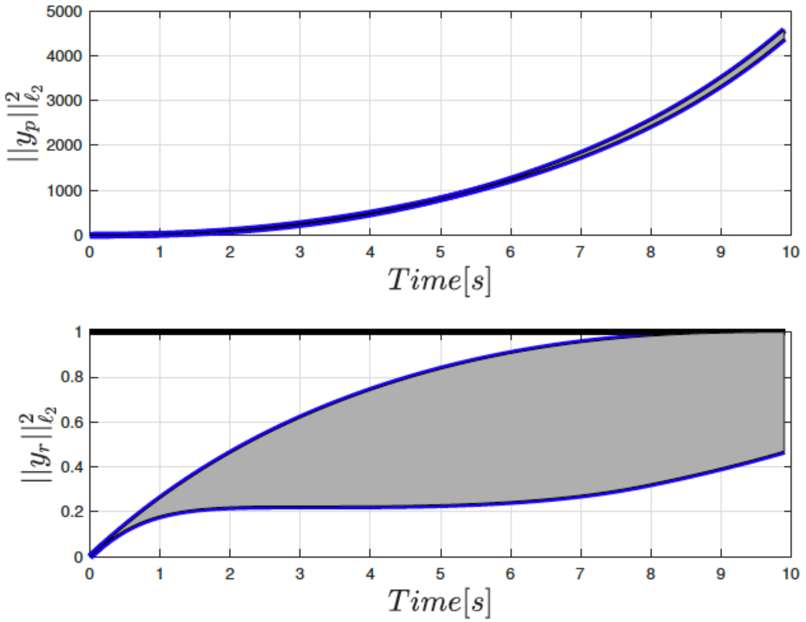
The attack signal must be stealthy for all sampled realizations. Very likely to be bounded!



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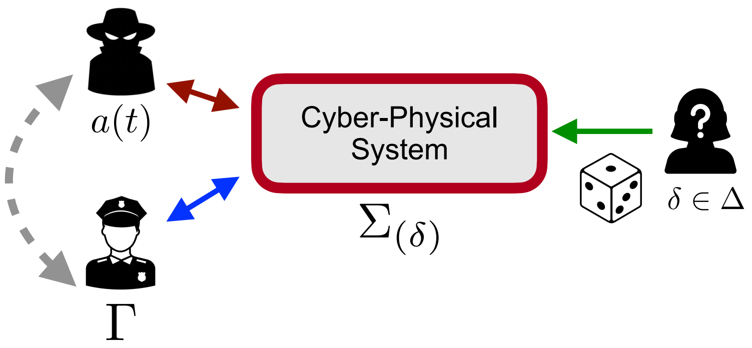

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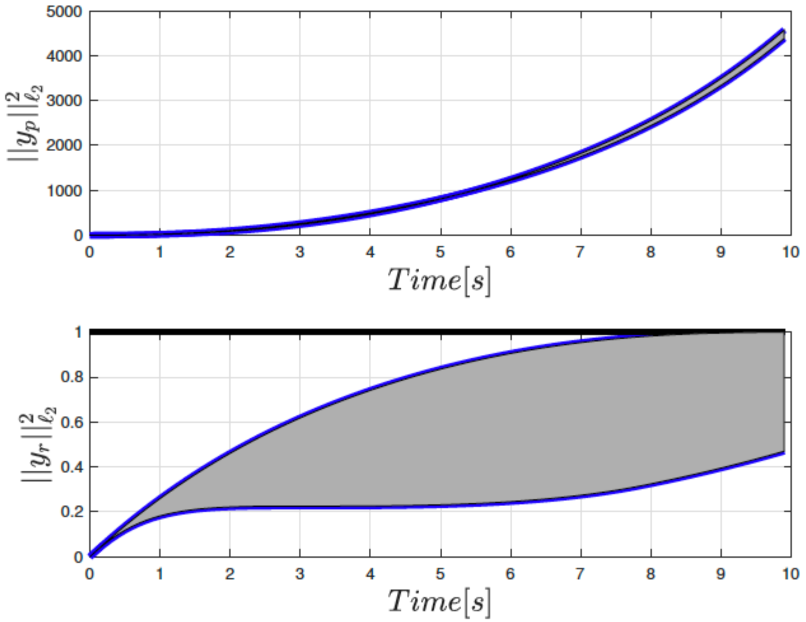

Risk-optimal defense:

$$\min_{\Gamma} R_{\hat{\Delta}_N}(\Gamma)$$

Anand et al.. "Risk Assessment of Stealthy Attacks on Uncertain Control Systems". IEEE TAC, 2023

Gallo et al. "Design of multiplicative watermarking against covert attacks". CDC 2021

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- **Conclusions and Remarks**

Conclusions and Remarks

- Risk management is a more comprehensive term than *security*.
- The importance of Adversary models to define (in)security.
- Security metrics - a bridge between risk management and game-theoretic design
- The role of Uncertainty and its relation to the Adversary.
    - Worst-case allows colluding with Adversary
    - Omniscient vs Bounded-Rationality
    - Uncertainty can be used as a form of defense (MTD)

Acknowledgments

Swedish Research Council

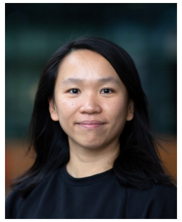
Swedish Foundation *for* Strategic Research

Uppsala Secure Learning and Control Lab

USLC

https://uslc-lab.github.io/

TU Delft



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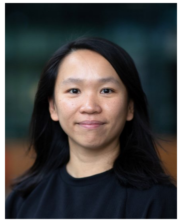

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We are hiring!

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Knut and Alice Wallenberg Foundation

Swedish Research Council

Swedish Foundation for Strategic Research

VINNOVA
Sweden's Innovation Agency

Swedish Energy Agency

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Swedish
Research Council

Swedish Foundation for Strategic Research

Sweden's Innovation Agency

Swedish Energy Agency

Openings:

- Hiring 1 Postdoc in Secure Federated Learning
- Hiring 1 PhD student in Distributed Voltage Control
- More positions to come in 2024/2025!

Uppsala Universitet

Backup slides

Dissipative Systems Theory

Consider the LTI system Σ with input a and outputs y_p and y_r. The following statements are equivalent:

1. the system Σ is dissipative w.r.t. $s(a,x) = \beta\|y_r(t)\|_2^2 - \|y_p(t)\|_2^2$;
2. for all trajectories of the system such that $T > 0$ and $x(0) = 0$, we have $\int_0^T \|y_p(t)\|_2^2 \leq \beta \int_0^T \|y_r(t)\|_2^2$;
3. there exists a positive semi-definite matrix $P \succeq 0$ such that the following linear matrix inequality (LMI) holds:

$$\begin{bmatrix} A^\top P + PA & PB \\ B^\top P & 0 \end{bmatrix} - \beta \begin{bmatrix} C_r^\top C_r & C_r^\top D_r \\ D_r^\top C_r & D_r^\top D_r \end{bmatrix} + \begin{bmatrix} C_p^\top C_p & C_p^\top D_p \\ D_p^\top C_p & D_p^\top D_p \end{bmatrix} \preceq 0.$$

J.C. Willems, “Dissipative dynamical systems Part II: Linear systems with quadratic supply rates”, Archive for Rational Mechanics and Analysis, 45 (5) (1972), pp.352-393

H.L. Trentelman, J.C. Willems, “The Dissipation Inequality and the Algebraic Riccati Equation”. In: Bittanti S., Laub A.J., Willems J.C. (eds) The Riccati Equation. Communications and Control Engineering Series. Springer, Berlin, Heidelberg (1991)

Dissipative Systems Theory

$$\gamma^\star = \min_{\beta \geq 0} \quad \beta$$
$$\text{s.t.} \quad \beta \|y_r\|^2_{\mathcal{L}_2} - \|y_p\|^2_{\mathcal{L}_2} \geq 0, \ \forall\, a \in \mathcal{L}_{2e}, \ x(0) = 0$$

Consider the LTI system Σ with input a and outputs y_p and y_r. The following statements are equivalent:

1. the system Σ is dissipative w.r.t. $s(a, x) = \beta \|y_r(t)\|_2^2 - \|y_p(t)\|_2^2$;
2. for all trajectories of the system such that $T > 0$ and $x(0) = 0$, we have $\int_0^T \|y_p(t)\|_2^2 \leq \beta \int_0^T \|y_r(t)\|_2^2$;
3. there exists a positive semi-definite matrix $P \succeq 0$ such that the following linear matrix inequality (LMI) holds:

$$\begin{bmatrix} A^\top P + PA & PB \\ B^\top P & 0 \end{bmatrix} - \beta \begin{bmatrix} C_r^\top C_r & C_r^\top D_r \\ D_r^\top C_r & D_r^\top D_r \end{bmatrix} + \begin{bmatrix} C_p^\top C_p & C_p^\top D_p \\ D_p^\top C_p & D_p^\top D_p \end{bmatrix} \preceq 0.$$

J.C. Willems, "Dissipative dynamical systems Part II: Linear systems with quadratic supply rates", Archive for Rational Mechanics and Analysis, 45 (5) (1972), pp.352-393

H.L. Trentelman, J.C. Willems, "The Dissipation Inequality and the Algebraic Riccati Equation". In: Bittanti S., Laub A.J., Willems J.C. (eds) The Riccati Equation. Communications and Control Engineering Series. Springer, Berlin, Heidelberg (1991)

⚠ *Note of caution:* in general, there is no simple equivalent frequency domain inequality

H.L. Trentelman. When does the algebraic Riccati equation have a negative semi-definite solution?. In: Blondel, V., Sontag, E.D., Vidyasagar, M., Willems, J.C. (eds) Open Problems in Mathematical Systems and Control Theory. Communications and Control Engineering. Springer (1999)

Classical fault-tolerant control design objectives

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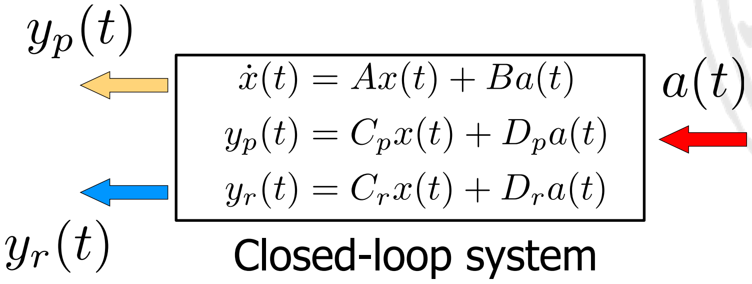
- Both are based on *sensitivity metrics*:
  - **Robustness**: largest **impact on performance** of <u>unit-energy</u> faults
  - **Detectability**: smallest **detectability** of <u>unit-energy</u> faults

Classical Sensitivity Metrics

$\mathcal{L}_{2e} =$ "signals with finite energy over finite time intervals"

$$\|y\|_{\mathcal{L}_2}^2 \triangleq \int_{-\infty}^{+\infty} \|y(t)\|_2^2 \, dt$$



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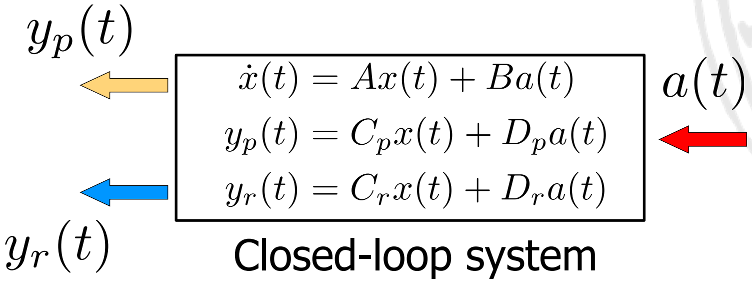

- Robustness:

$$\begin{aligned} \gamma_{H_\infty} \triangleq \sup_{a \in \mathcal{L}_{2e}} \quad & \|y_p\|_{\mathcal{L}_2} \\ \text{s.t.} \quad & \|a\|_{\mathcal{L}_2} = 1 \\ & x(0) = 0 \end{aligned}$$

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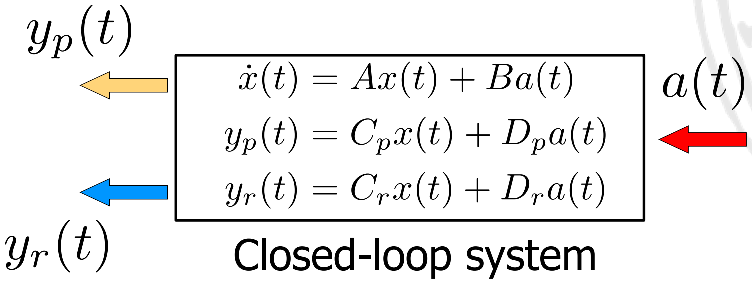

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$$\gamma_{H_\infty} = \sup_{w \geq 0} \bar{\sigma}_p(jw)$$

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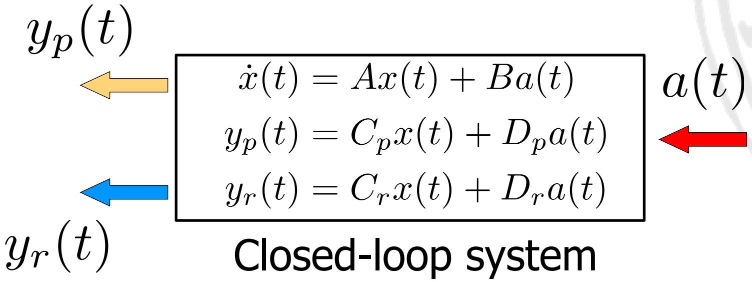

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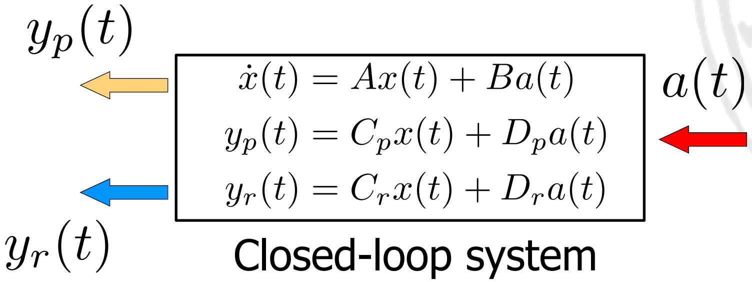

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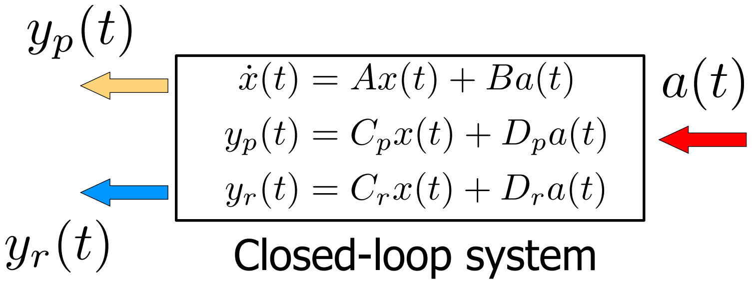
Classical Sensitivity Metrics



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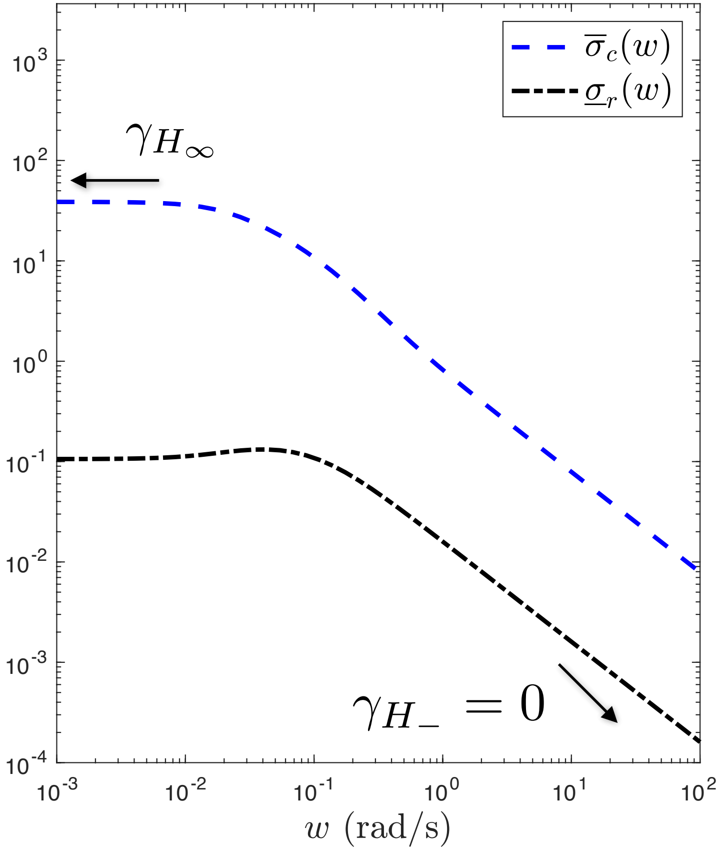

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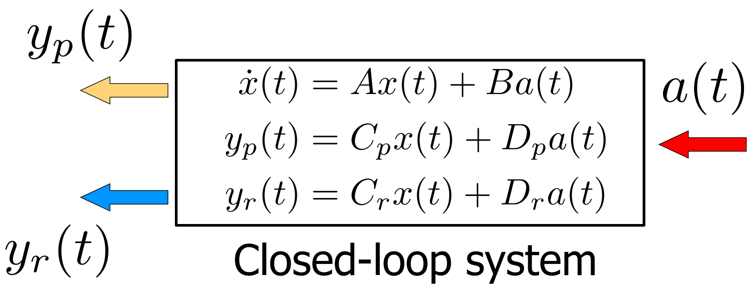

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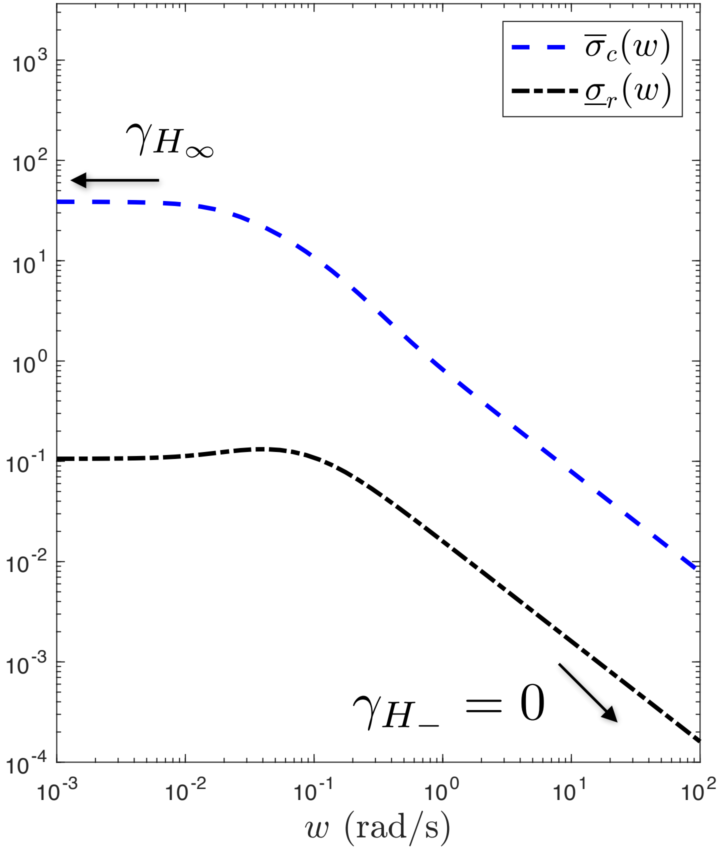

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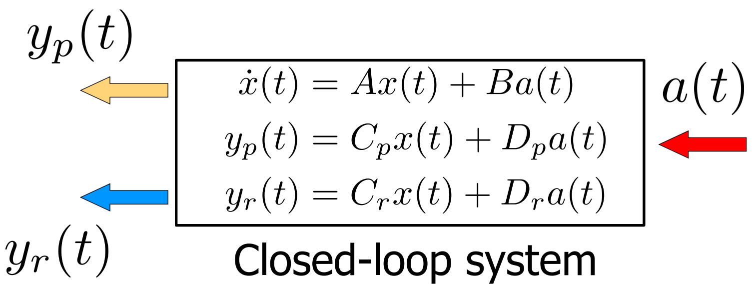

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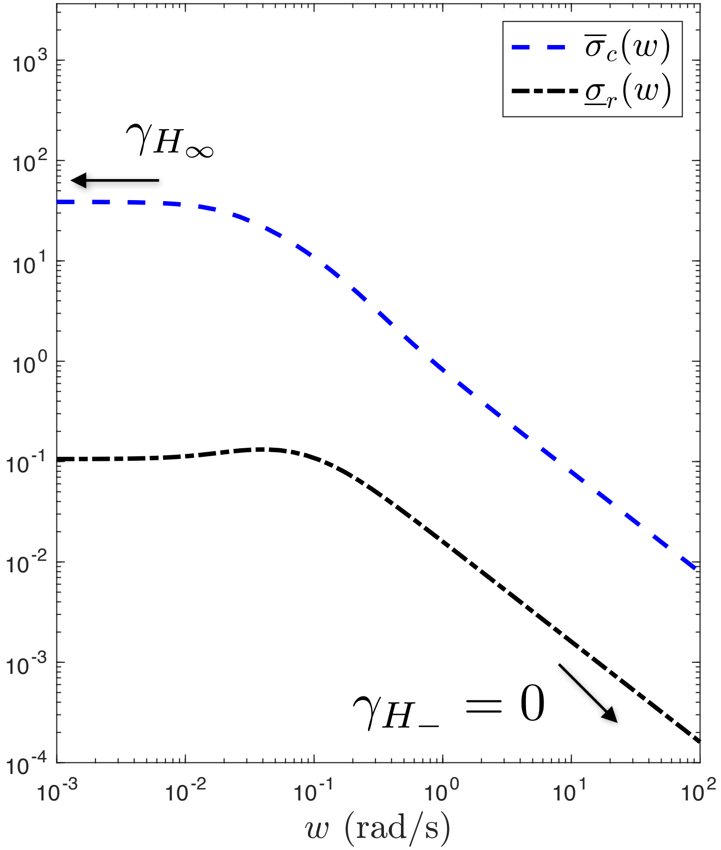

- **Limitation of mixed metrics:** worst-case frequency is not the same
  - Each metric looks at **different** worst-case inputs!

Example: Robust Stealthy Attacks

Anand et al.. “Risk Assessment of Stealthy Attacks on Uncertain Control Systems”. IEEE TAC, 2023

SYSTEM PARAMETERS

K_{lm}	1	T_{lm}	6
T_g	0.2	R	0.05
T_h	[4 6]	T_s	0.1

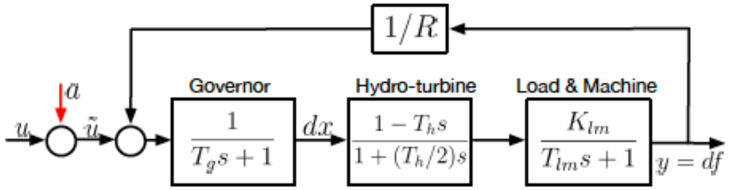

Fig. 4. Power generating system with a hydro turbine.

Gallo et al. “Design of multiplicative watermarking against covert attacks”. CDC 2021

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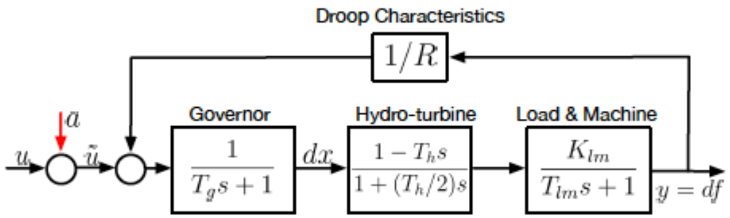

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Without uncertainty:

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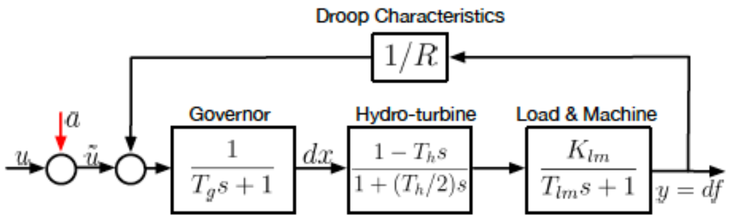

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Without uncertainty:

- Unbounded impact for any parameter value $T_h \in [4{,}6]$.

With uncertain T_h:

- Impact becomes bounded when T_h is uncertain & attack is robust
- “Uncertainty as a defense” can be incorporated by design
  - Watermarking, moving target, weak encryption, …

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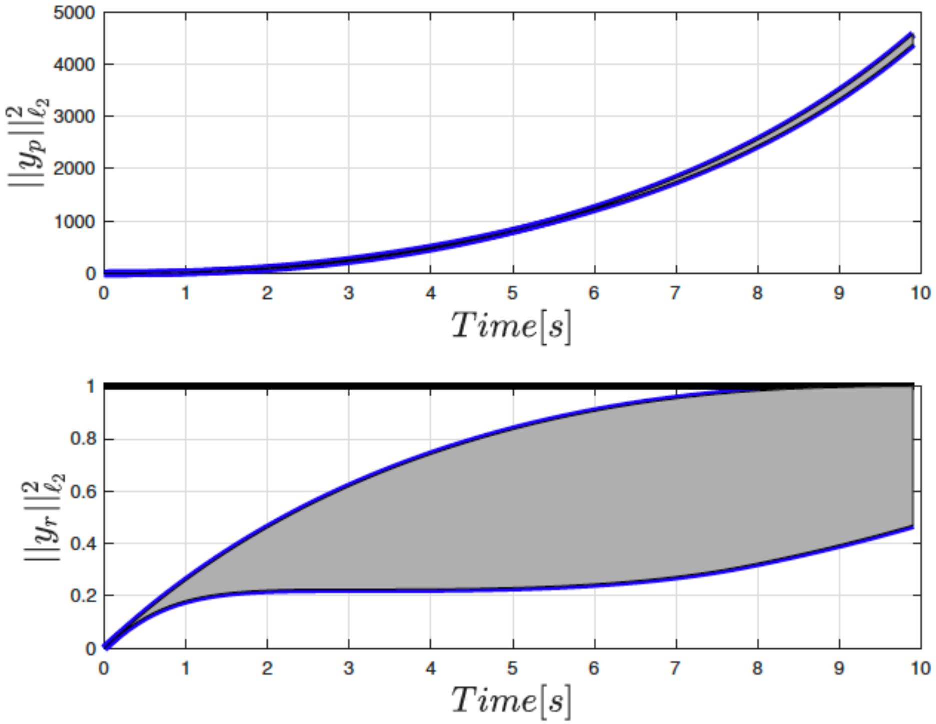

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