Networked Control Systems under Cyber Attacks with Applications to Power Networks

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Outline

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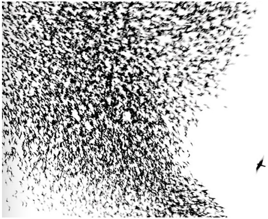
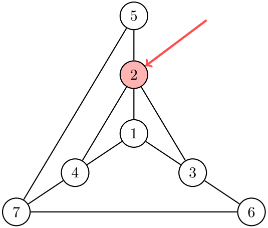
1. Introduction
   - Motivation
   - Fault Detection and Isolation
2. The Consensus Protocol
   - Consensus
   - Consensus in NMAS under Attack on Node
   - Consensus in NMAS under Communication Attacks
   - Reducing the Number of Monitoring Nodes
3. Power Systems
   - Classical Model

- Several agents interacting with each other
  - Information exchange or physical coupling
- Cooperation needed to achieve common goal
- Only local information available
  (*i.e.* from neighbors)
- Decentralized / Distributed Controllers

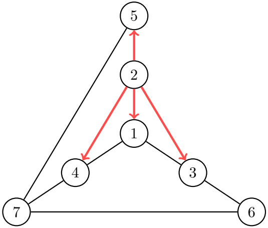


- What happens to the entire network if a single agent misbehaves?
- How can the other agents detect the misbehavior?
- Can the misbehaving node be identified?
- How should the network react?

Attack on Node



Attack on Communications



- How to detect and identify the misbehaving node in a distributed fashion?
- How to distinguish between an attack on a node and an attack on the communications?

- Dynamics of node k under attack in k

$$\dot{x}_k = A_{kk}x_k + \sum_{j \neq k} A_{kj}x_j + f_k$$

- Global dynamics seen from i under attack in k

$$\begin{cases} \dot{\mathbf{x}} = A\mathbf{x} + b_f^k f_k \\ \mathbf{y}_i = C_i\mathbf{x}, \end{cases}$$

where

- $\mathbf{y}_i$ are the measurements available at node i.
- b_f^k is the attack signature
- C_i is a design parameter

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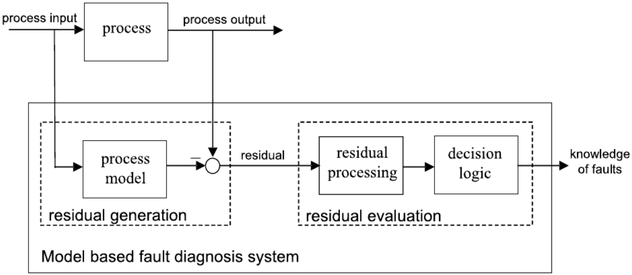

- Basic Ideas:
  - Compute an expected output;
  - Compare and evaluate the real and expected outputs.

Model-based Fault Detection and Isolation

Generalized Observer Scheme

- Implement a Generalized Observer Scheme (GOS) based on a bank of observers such that:
  - Each observer i is insensitive to only one fault element, f_i
  - The residual r_i is then sensitive to all faults except f_i
  - The fault f_i is detected using the following threshold logic:

$$\begin{cases} \|r_i(t)\| < T_{f_i} \\ \|r_k(t)\| \geq T_{f_k} \;, \forall k \neq i \end{cases}$$

Example

Let $f \in \mathbb{R}^3$. Build a bank of 3 observers according to the GOS.

	f_1	f_2	f_3
$\|r_1\|$	0	$+$	$+$
$\|r_2\|$	$+$	0	$+$
$\|r_3\|$	$+$	$+$	0

Unknown Input Observer

- Consider the faulty system:

$$\begin{cases} \dot{\mathbf{x}} = A\mathbf{x} + b_f^k f_k \\ \mathbf{y}_i = C_i\mathbf{x} \end{cases}$$

Definition

A state observer is an unknown input observer (UIO), with respect to f_k, if the state estimation error $e_i^k = \mathbf{x} - \hat{\mathbf{x}}_i^k$ approaches zero asymptotically, regardless of the presence of an unknown input f_k.

- Such UIO for the previous perturbed system has the following dynamics:

$$\begin{cases} \dot{z} = Fz + TBu + K\mathbf{y}_i \\ \hat{x}_i^k = z + H\mathbf{y}_i \end{cases}$$

- Choose the matrices F, T, K, H to satisfy the following conditions:

$$\begin{aligned} F &= A - HC_iA - K_1C_i \\ T &= I - HC_i \\ (HC_i - I)\, b_f^k &= 0 \\ K_2 &= FH \\ K &= K_1 + K_2 \end{aligned}$$

Theorem

The necessary and sufficient conditions for this UIO to exist are:

$$rank\left(C_i b_f^k\right) = rank\left(b_f^k\right) = 1, \quad rank\left(\begin{bmatrix} sI_n - A & b_f^k \\ C_i & 0 \end{bmatrix}\right) = n + 1$$

for all $Re(s) \geq 0$.

- Estimation error's dynamics and residual when all faults are active

$$
\begin{aligned}
\dot{e}_i^k &= F e_i^k + (I - HC_i) B_f^{-k} f_{-k} \\
r_i^k &= C_i e_i^k
\end{aligned}
$$

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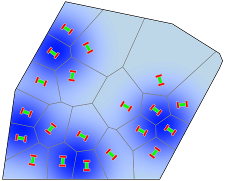


- The main objective of such protocol is to achieve an agreement on a certain quantity of interest
- Example of applications:
  - Rendezvous
  - Formation
  - Deployment
  - Load balancing
  - Distributed estimation

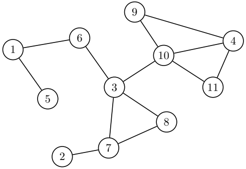
- Agents with single integrator dynamics:

$$\begin{cases} \dot{x}_i = u_i & , \; x_i(0) = x_{i_0} \in \mathbb{R} \\ y_i = x_i \end{cases}$$

- Distributed control law given by:

$$u_i = -\sum_{j \in N_i} (y_i - y_j)$$

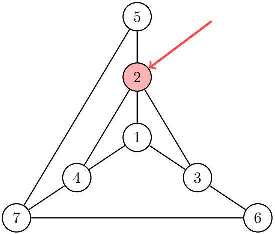
  - Based on local information only
  - Relies on the information transmitted by the neighbors, y_j



- Global dynamics of the network:

$$\dot{\mathbf{x}} = -\mathcal{L}\mathbf{x} \qquad (1)$$

with
$\mathbf{x} = \left[x_1^T \cdots x_N^T\right]^T$



- Dynamics of the attached node k:

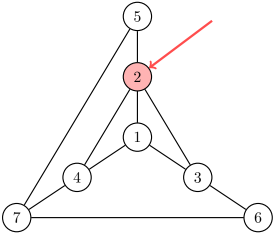
$$\begin{cases} \dot{x}_k & = & -\sum_{j \in N_k} (y_k - y_j) + f_k \\ y_k & = & x_k \end{cases}$$

- Global dynamics of the network:

$$\dot{\mathbf{x}} = -\mathcal{L}\mathbf{x} + b_f^k f_k \qquad (2)$$

  - $b_f^k \in \mathbb{R}^N$ is a vector with the k^{th} component set to 1 and all the others to 0

- The same form as $\dot{x}(t) = Ax(t) + b_f^k f(t)$



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Assumption

The graph of the network is known by all nodes and it remains constant.

- Distributed scheme:
  - Have each node monitoring all its neighbors using a GOS
- Information available at node i is

$$\mathbf{y}_i = \begin{bmatrix} y_i^T & y_{i_1}^T & \cdots & y_{i_{|N_i|}}^T \end{bmatrix}^T = \begin{bmatrix} x_i^T & x_{i_1}^T & \cdots & x_{i_{|N_i|}}^T \end{bmatrix}^T = C_i \mathbf{x}$$

- For each neighbor k, design a UIO for the global dynamics insensitive only to an attack on node k

$$\begin{cases} \dot{z}_i^k & = & F_i^k z_i^k + K_i^k \mathbf{y}_i \\ \hat{\mathbf{x}}_i^k & = & z_i^k + H_i^k \mathbf{y}_i \end{cases} \qquad (3)$$

- Reminding the necessary and sufficient conditions for the UIO
  1. $\text{rank}\left(C_i b_f^k\right) = \text{rank}\left(b_f^k\right) = 1$
  2. The transmission zeros of $\left(-\mathcal{L}, b_f^k, C_i, 0\right)$ are stable
- Derived results:

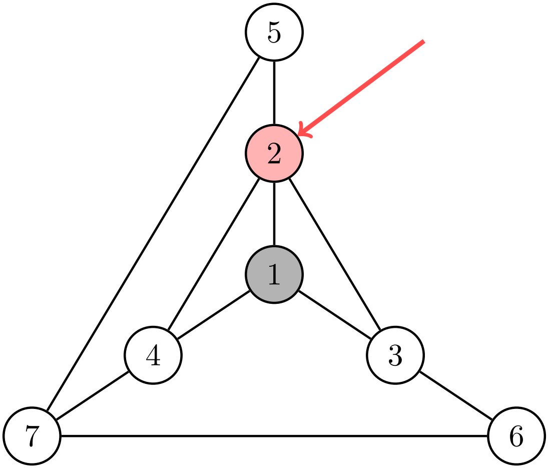
Lemma

If an undirected graph $\mathcal{G}$ is connected, then any principle minor of its Laplacian matrix $\mathcal{L}$, induced by a subset of nodes $\bar{F} \subset \mathcal{V}$, is invertible.

Theorem

There exists a UIO for the system $\left(-\mathcal{L}\left(\mathcal{G}\right), b_f^k, C_i, 0\right)$ if the graph $\mathcal{G}$ is connected and $k \in \mathcal{N}_i$.

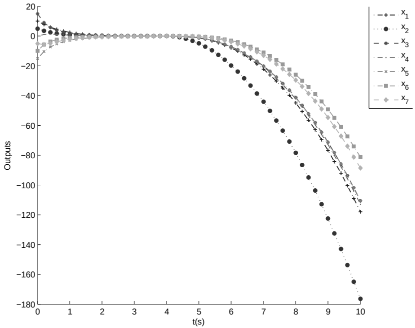
- Attack in node 2 seen from node 1



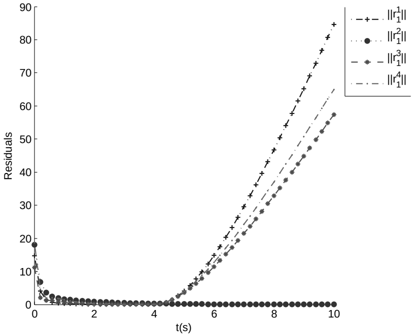
Consensus in NMAS under Attack on Node

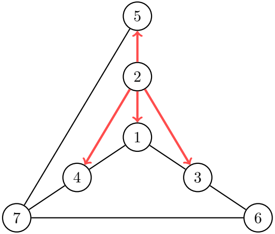
Results 1)

Outputs



Residuals at node 1





- Dynamics of the compromised node k:

$$\begin{cases} \dot{x}_k &= -\sum_{j \in N_k} (w_k - y_j) \\ w_k &= x_k \\ y_k &= x_k + f_k \end{cases}$$

  - w_k is an internal measurement of the state, not being subject to an attack on the communications

- Global dynamics of the network:

$$\begin{cases} \dot{\mathbf{x}} &= -\mathcal{L}\mathbf{x} + \mathcal{I}_{\bar{k}} l^k f_k \\ \mathbf{y} &= \mathbf{x} + b_f^k f_k \\ \mathbf{w} &= \mathbf{x} \end{cases} \tag{4}$$

- Separating the dynamics of the healthy network $\bar{k}$

$$\begin{cases} \dot{\mathbf{x}}_{\bar{k}} = -\mathcal{L}_{\bar{k}}\mathbf{x}_{\bar{k}} - l_{\bar{k}k}y_k \\ \mathbf{y}_{\bar{k}} = \mathbf{x}_{\bar{k}} \end{cases} \tag{5}$$

- Note that y_k is the information transmitted by node k
  - Attack in node k:

$$\begin{cases} \dot{x}_k = -\mathcal{L}_k x_k - l_{k\bar{k}}\mathbf{y}_{\bar{k}} + f_k \\ y_k = x_k \end{cases}$$

  - Communication attack in node k:

$$\begin{cases} \dot{x}_k = -\mathcal{L}_k x_k - l_{k\bar{k}}\mathbf{y}_{\bar{k}} \\ y_k = x_k + f_k \end{cases}$$

- The "healthy" network can not distinguish between both attacks

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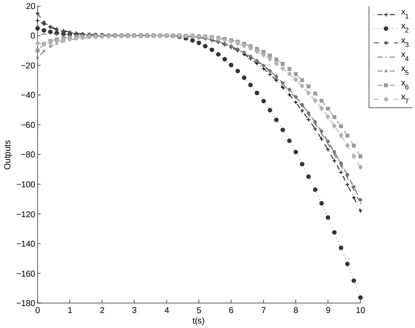
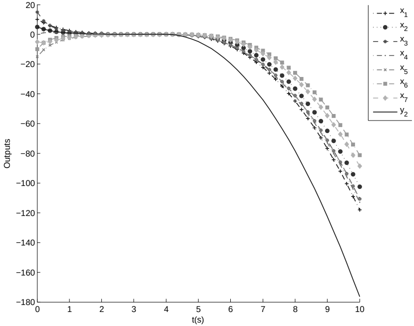
Attack in node 2



Communication attack in node 2

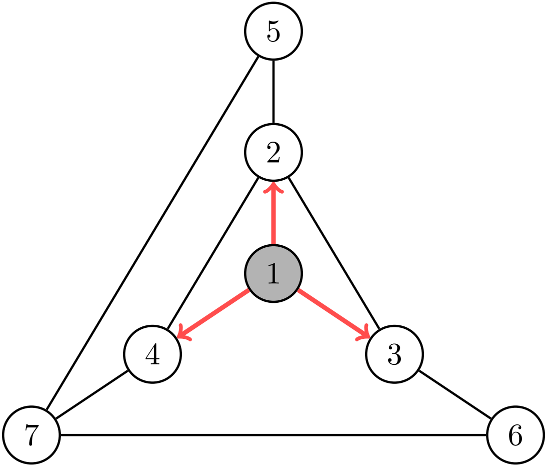


- Key observations:
  - Node k followed the rest of the network under the communication attack
  - Thus it should be able to realize something is wrong
- For node k, it seems all its neighbors are misbehaving in a particular way
- Consider the previous system monitored from node k

$$\begin{cases} \dot{\mathbf{x}} = -\mathcal{L}\mathbf{x} + b_f^k f_k \\ \mathbf{y}_k = C_k \mathbf{x} \end{cases} \tag{6}$$

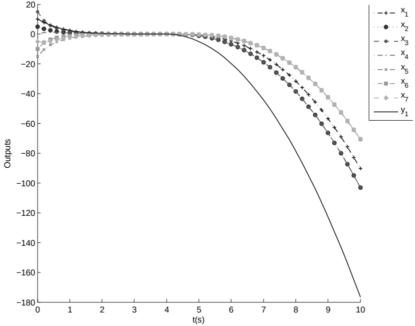
  - with $b_f^k = \mathcal{I}_{\bar{k}} l^k$
  - and $\mathbf{y}_k = \begin{bmatrix} w_k & y_{k_1} & \cdots & y_{k_{|\mathcal{N}_k|}} \end{bmatrix}^T$
- Add an UIO insensitive to b_f^k to the observer bank in node k

- Attack in node 1 seen from node 1



Consensus in NMAS under Communication Attacks
Results 2)

Outputs



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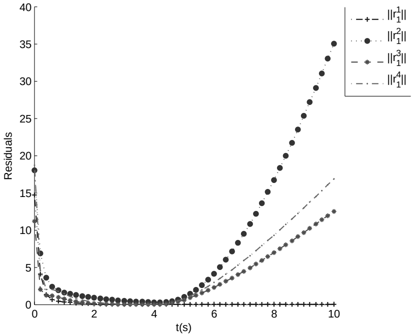
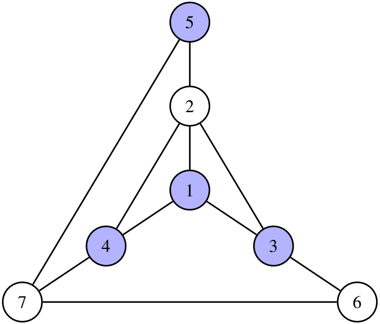


Reducing the Number of Monitoring Nodes

- The problem of reducing the number of observers is related to the set cover:

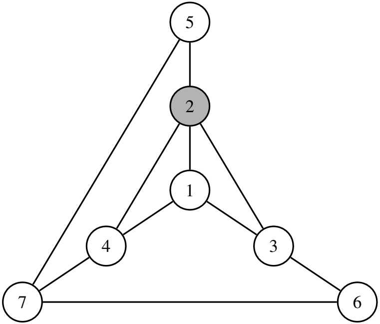
$$\min_{S \subseteq \mathcal{V}} \quad |S|$$
$$\text{s.t.} \bigcup_{i \in S} N_i = \mathcal{V}$$

- Each observer node is monitored by at least one other node.

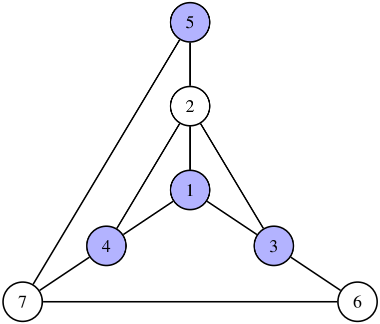


Reducing the Number of Monitoring Nodes

Monitoring Nodes



Observed Nodes



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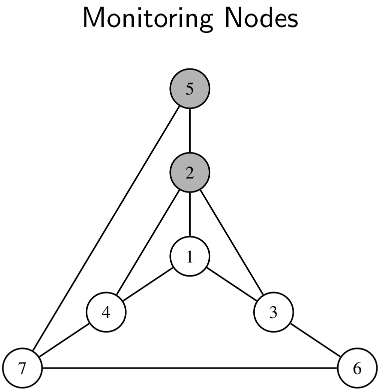

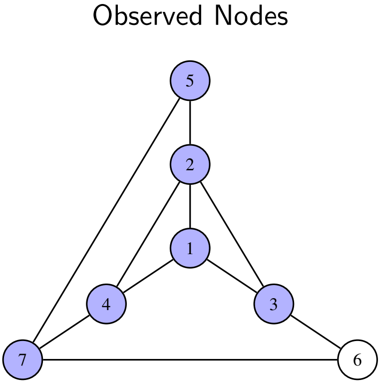

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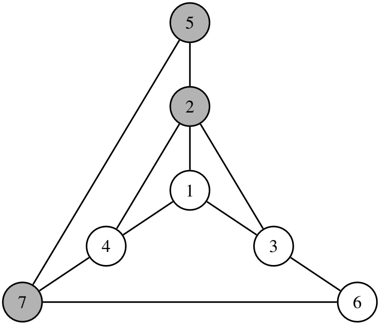

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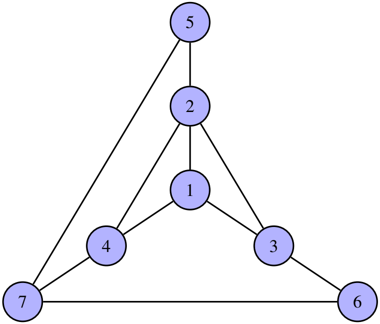

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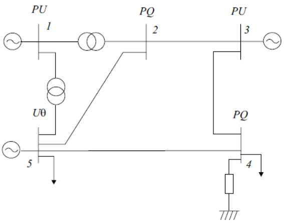
- Active power flow on a loss-less distribution grid.
- Each bus has dynamics given by the "swing equation":

$$M_i\ddot{\delta}_i + D_i\dot{\delta}_i = -\sum_{j\in N_i} w_{ij}\sin(\delta_i - \delta_j) + P_{m_i}$$



- $\delta_{ij} = \delta_i - \delta_j$ is small, thus $\sin(\delta_i - \delta_j) \approx \delta_i - \delta_j$.

It can be looked at from a multi-agent systems point of view.

- consider δ_i and $\dot{\delta}_i(t)$ to be states of each bus.
- Having $x = [\delta_1, \ldots, \delta_N, \dot{\delta}_1, \ldots, \dot{\delta}_N]$: $\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{P}_m$.

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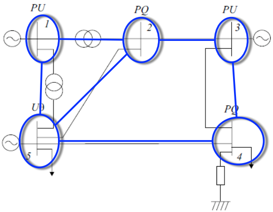

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- Existence of UIO:

Theorem

There exists an UIO for the system $\left(A, b_f^k, C_i, 0\right)$ *if the graph* $\mathcal{G}$ *is connected,* k *is a neighbor of* i *and node* i *measures both the phase-angle and the frequency offset of its neighbors.*

- Infeasibility results:

Theorem

Let the graph $\mathcal{G}$ *be connected and* k *be a neighbor of* i*. No UIO for the system* $\left(A, b_f^k, C_i, 0\right)$ *exists if node* i *only measures either the phase-angle or the frequency offset of its neighbors.*

Summary

- Distributed techniques to detect and isolate attacks on nodes and communication attacks in a network of agent using the *consensus protocol* were proposed and sufficient conditions were also provided
- It was shown that the "healthy network" can not distinguish between the two types of attack, but the misbehaving node can
- A distributed FDI scheme for power systems was proposed